21

Inequalities

By studying this lesson you will be able to;

- solve inequalities of the form $x \pm a \gtreqless b$,
- solve inequalities of the form $ax \gtreqless b$,
- find the integral solutions of an inequality,
- represent the solutions of an inequality on a number line.

In Sri Lanka, a person who is 55 years or older is considered to be a senior citizen. Accordingly, if we denote the age of a senior citizen by t, then we can indicate this by the inequality $t \geqslant 55$. This means that the value of t is always greater or equal to 55.

Let us recall what was learnt in grade 8 regarding inequalities.

$x > 3$ is an inequality. This means that the values that x can take are the values that are greater than 3. However, if we write $x \geq 3$, this means that the values that x can take are 3 or a value greater than 3.

Similarly, $x < 3$ means that the values that x can take are the values that are less than 3, and $x \leq 3$ means that the values that x can take are 3 or a value less than 3.

For example, the solution set of $x > 3$ is the set of all numbers which are greater than 3. The set of integral solutions of this inequality is $\{4, 5, 6, 7, \ldots\}$.

The three dots mean that all the integers which belong to the pattern indicated by the first few values belong to the solution set. Therefore, the above inequality has infinitely many integral solutions.

Although in mathematics it is important to represent all the solutions as a set, when indicating the integral solutions of an inequality, it is sufficient to just write the values. For example, the integral solutions of the inequality $x > 3$ can be written as 4, 5, 6, … .

The solution set of an inequality which has an algebraic term, is the set of all values that the algebraic term can take. Let us recall what has been learnt earlier regarding the solution set of an inequality, and how this is represented on a number line, by considering the following examples.

Example 1

Consider the inequality $x > 3$.

i. Write the set of **integral solutions** of the above inequality.
ii. Represent the **integral solutions** of the above inequality on a number line.

i. $\{4, 5, 6, 7, 8, ...\}$

ii.

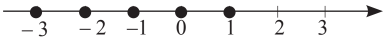
Example 2

Consider the inequality $x \leq 1$.

i. Write the set of integral solutions of the above inequality.
ii. Represent the integral solutions of the above inequality on a number line.

i. $\{..., -3, -2, -1, 0, 1\}$

ii.



Example 3

Represent the solution set of the inequality $x > -3\frac{1}{2}$ on a number line.

Example 4

Represent the solution set of the inequality $x \geq -2$ on a number line.

Example 5

Represent the **solution set** of the inequality $-3 < x \leq 3\frac{1}{2}$ on a number line.

Review Exercise

1. For each of the following inequalities, represent the **set of integral solutions on** a number line.

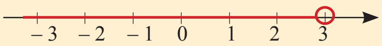
i. $x > 2$ **ii.** $x \geq -1$ **iii.** $x < 4$ **iv.** $x \leq -2.5$ **v.** $x > 1\frac{1}{2}$

2. For each of the following, write an inequality that has the values represented on the number line as its solution set.

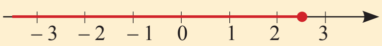
i.

ii.

iii.



iv.



3. Represent the solution set of each of the following inequalities on a number line.

i. $-1 < x < 2$ **ii.** $-2 \leq x < 3$ **iii.** $-3 < x \leq 1$

iv. $x < -1$ or $x \geq 2$ **v.** $x \leq -3$ or $x > 0$

21.1 Inequalities of the form $x \pm a \gtrless b$

The following notice has been placed near a certain bridge.

"This bridge can only bear loads of less than 10 tons"

Suppose a lorry of mass 4 tons carrying a certain load wishes to cross this bridge. If we take the mass of the load carried by the lorry to be x tons, then the lorry can safely cross the bridge only if $x + 4 < 10$. In other words, if the mass of the load carried by the lorry is x tons, then the lorry can safely cross the bridge, only if the inequality $x + 4 < 10$ is satisfied.

We can find the mass of the load that the lorry can safely carry across the bridge by solving the inequality $x + 4 < 10$.

Solving an inequality means, obtaining an inequality equivalent to the given inequality such that only x (or the given variable) is on one side of the inequality.

When solving an inequality, we can adopt the procedure followed in solving equations to a large extent.

For example, we can subtract 4 from both sides of the above inequality $x + 4 < 10$. Accordingly,
$x + 4 - 4 < 10 - 4$.
When we simplify this we obtain
$x < 6$.

Therefore, for the lorry to safely cross the bridge, the load that it carries should be less than 6 tons.

Example 1

Solve the inequality $x + 2 < 7$ and represent the integral solutions on a number line.

$x + 2 < 7$

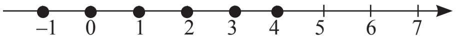
$x + 2 - 2 < 7 - 2$ (subtracting 2 from both sides)

$\underline{\underline{x < 5}}$

The integral solutions of this inequality are the integers that are less than 5. That is, the values 4, 3, 2, 1, 0, –1, –2, … .

These integral solutions can also be expressed as a set as $\{4, 3, 2, 1, 0, -1, -2, \ldots\}$
The solution set can be represented on a number line as follows.

Integral solutions of x



Example 2

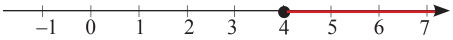
Solve the inequality $x - 3 \geq 1$ and represent the solution set on a number line.

$x - 3 \geq 1$

$x - 3 + 3 \geq 1 + 3$ (adding 3 to both sides)

$\underline{\underline{x \geq 4}}$

Now let us represent the solution set on a number line.



All the solutions (numbers that are greater or equal to 4) are represented here. It is important to remember that not only the integral solutions, but solutions such as 4.5 and 5.02 are also included.

Example 3

The maximum mass that a bag can carry is 6 kg. Nimal puts x packets of rice of mass 1 kilogramme each and 2 packets of sugar of mass 1 kilogramme each into this bag. This information can be represented by the inequality $x + 2 \leq 6$.

i. Solve this inequality.

ii. What is the maximum number of packets of rice that Nimal can carry in this bag?

i. $x + 2 \leq 6$

$x + 2 - 2 \leq 6 - 2$

$\underline{\underline{x \leq 4}}$

ii. Therefore, the maximum number of packets of rice that can be carried in this bag is 4.

Exercise 21.1

1. Solve each of the following inequalities and write the set of integral solutions.

i. $x + 3 > 5$
ii. $x - 4 < 1$
iii. $x - 7 \geq - 6$
iv. $2 + x \leq - 4$
v. $7 + x > 5$

2. Solve each of the following inequalities and represent the solution set on a number line.

i. $x + 1 > 3$
ii. $x - 3 \leq 1$
iii. $6 + x \geq 2$
iv. $x - 7 < - 7$
v. $x + 5 > - 1$

3. Sakindu has 60 rupees. He buys a book for x rupees and a pen for 10 rupees. The total value of the items he bought can be expressed in terms of an inequality as $x + 10 \leq 60$. Solve this inequality and determine the maximum price that the book could be.

4.The maximum number of people that can travel in a certain van is 15. If 3 people get into the van from one location and x number of people from another location, this information can be represented by the inequality $x + 3 \leq 15$.

i. Solve the above inequality.

ii. What is the maximum number of people that can get into the van from the second location?

5. The sum of the ages of Githmi and Nethmi does not exceed 30. Githmi is 14 years old. If Nethmi's age is taken as x years, this information can be represented by the inequality $x + 14 \leq 30$. Solve this inequality and find the maximum age that Nethmi could be.

21.2 Inequalities of the form $ax \gtrless b$

The price of two books of the same type is more than 40 rupees. If we take the price of one of these books as x rupees, we can represent this information by the inequality $2x > 40$ involving x. By solving this inequality, we can find the price that each book could be.

When solving this type of inequalities there are some important facts that we should keep in mind.

Consider the following inequalities.

i. The inequality $3 < 4$ is true.
$2 \times 3 < 2 \times 4$ (multiplying both sides by 2)
The inequality $6 < 8$ is true.

ii. The inequality $8 > 6$ is true.
$\frac{8}{2} > \frac{6}{2}$ (dividing both sides by 2)
The inequality $4 > 3$ is true.

When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.

iii. The inequality $2 < 3$ is true.
$2 \times -2 < 3 \times -2$ (multiplying both sides by -2)
The inequality $-4 < -6$ which is obtained is false. However, $-4 > -6$ is true.

iv. The inequality $9 > 6$ is true.
$\frac{9}{-3} > \frac{6}{-3}$ (dividing both sides by -3)
The inequality $-3 > -2$ which is obtained is false. However, $-2 < -3$ is true.

When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc .

Through the following examples, let us learn how inequalities are solved taking into consideration the above facts.

Example 1

Solve the inequality $2\,x < 12$ and represent the solutions on a number line.

$$2\,x < 12$$

$$\frac{2x}{2} < \frac{12}{2} \text{ (dividing both sides by 2)}$$

$$\underline{\underline{x < 6}}$$

Example 2

Solve the inequality $3x \geq 12$.

$$3x \geq 12$$

$$\frac{3x}{3} \geq \frac{12}{3}$$

$$\underline{\underline{x \geq 4}}$$

Example 3

Solve the inequality $-5x \leq 15$.

$$-5x \leq 15$$

$$\frac{-5x}{-5} \geq \frac{15}{-5} \text{ (when dividing by a negative number the sign changes)}$$

$$\underline{\underline{x \geq -3}}$$

Example 4

Solve the inequality $\frac{x}{3} < 2$.

$$\frac{x}{3} \times 3 < 2 \times 3 \qquad \text{(multiplying both sides by 3)}$$

$$\underline{\underline{x < 6}}$$

Example 5

Solve the inequality $-\frac{2x}{5} > 6$.

$$-\frac{2x}{5} > 6$$

$$-\frac{2x}{5} \times 5 > 6 \times 5 \quad \text{(multiplying both sides by 5)}$$

$$-2x > 30$$

$$\frac{-2x}{-2} < \frac{30}{-2} \quad \text{(the sign changes when dividing by} -2)$$

$$\underline{\underline{x < -15}}$$

Exercise 21.2

1. Solve each of the following inequalities and write the integral solutions.

i. $2x > 6$ **ii.** $3x \leq 12$ **iii.** $-5x \geq 10$ **iv.** $-7x < -35$

v. $-2x > -5$ **vi.** $\frac{x}{2} \leq 1$ **vii.** $\frac{x}{4} \geq -2$ **viii.** $-\frac{2x}{3} < 4$

2. Solve each of the following inequalities and represent the solutions on a number line.

i. $4x > 8$ **ii.** $7x \leq 21$ **iii.** $-3x \geq 3$ **iv.** $-2x < -6$

v. $\frac{x}{3} \geq 1$ **vi.** $\frac{x}{6} < -\frac{1}{6}$ **vii.** $\frac{2x}{3} \geq 4$ **viii.** $-\frac{3x}{5} < -\frac{1}{6}$

3. The price of 2 mangoes is less than 50 rupees. If the price of one mango is x rupees, this information can be represented by the inequality $2x \leq 50$. Solve this inequality and find the maximum possible price of a mango.

4. The maximum mass that can be carried by an elevator is 560 kilogrammes. Eight men of mass x kilogrammes each are riding this elevator. This information can be represented by the inequality $8x \leq 560$. Find the maximum mass that each man could be.

5.

(a) The amount of money Mahesh has is less than four times the amount that Ashan has. Mahesh as 68 rupees. If the amount that Ashan has is denoted by x rupees, then this information can be represented by the inequality $4x > 68$. Solve this inequality.

(b) If Ashan has only 5 rupee coins, what is the least amount he could be having?

Summary

- When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.
- When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc.

22

Sets

By studying this lesson you will be able to;

- identify finite and infinite sets,
- write the subsets of a given set,
- identify equivalent sets, equal sets, disjoint sets and universal sets,
- identify the intersection and union of two sets,
- identify the complement of a set,
- represent sets using Venn diagrams.

Introduction to sets

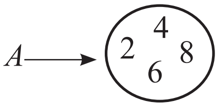
You have learnt earlier that a set is a collection of items that can be clearly identified. The items in a set are called its elements. Curly brackets are used to represent sets in terms of their elements. If a is an element of the set A, we denote this by $a \in A$. Moreover, the number of elements in the set A is denoted by $n(A)$.

Let us recall what was learnt previously about the different ways in which a set can be expressed.

1. Describing the set using a common characteristic by which the elements can be clearly identified
2. Writing the elements within curly brackets
3. Using a Venn diagram

As an example, let us consider how to write the set of all even numbers between 0 and 10 using the above 3 methods in the given order. Let us name this set A.

1. $A = \{\text{even numbers between 0 and 10}\}$
2. $A = \{2, 4, 6, 8\}$
3. A



The set which has no elements is known as the **null set**. The null set is denoted by { } and also by $\emptyset$. The number of elements in the null set is 0. Therefore, $n(\emptyset) = 0$.

Example 1

If P = {even prime numbers between 5 and 10}, then find $n(P)$.
Since there are no even prime numbers between 5 and 10, $P = \emptyset$ and therefore $n(P) = 0$.

Review Exercise

1. Determine whether each of the following collections is a set.

- **i.** Multiples of four between 0 and 30
- **ii.** The districts of Sri Lanka
- **iii.** Students who are good at mathematics
- **iv.** Triangular numbers
- **v.** The 10 largest integers

2. Write each set given below in terms of its elements and write the number of elements in each set too.

- **i.** A = {multiples of 5 between 0 and 20};
- **ii.** B = {letters of the word "RECONCILLIATION"}
- **iii.** C = {prime numbers between 2 and 13}
- **iv.** D = {numbers between 0 and 20 which are a product of two prime numbers}

3. D = {whole numbers between 5 and 10}.

- **i.** Write the elements of D.
- **ii.** Find $n(D)$.

4. Express the null set in three different ways using the above mentioned first method (That is, by the descriptive method using a special characteristic.)

22.1 Finite sets, infinite sets, equivalent sets and equal sets

Finite sets and infinite sets

Two sets which have been expressed in terms of a common characteristic by which the elements of the set can be clearly identified, are given below.

A = {multiples of 3 between 0 and 20}
B = {multiples of 5}

Let us write the elements of each set within curly brackets.

$A = \{3, 6, 9, 12, 15, 18\}$ $\qquad$ $B = \{5, 10, 15, 20, ...\}$

The number of elements in set A is 6. That is, the number of elements in this set is a specific number. Sets with a specific number of elements (that is, sets with a finite number of elements), are known as **finite sets**.

The number of elements in set B however, cannot be stated definitely. That is, the number of elements in this set is infinite. Three dots have been placed at the end of the list of numbers within curly brackets to denote that the set B has an infinite number of elements. Sets that have an infinite number of elements are known as **infinite sets**.

Example 1

For each of the sets given below, write the elements and write whether it is a finite set or an infinite set.

P = {positive multiples of 6 less than 30}
Q = {polygons}

$P = \{6, 12, 18, 24\}$ $\quad$ $n(P) = 4$
Q = {triangle, quadrilateral, pentagon, hexagon, … }

Since the number of elements in the set P is finite, P is a finite set. Since the number of elements in the set Q is infinite, Q is an infinite set.

Equal sets

Consider the two sets given below.

A = {even numbers between 0 and 10}

B = {digits of the number 48268}

These two sets can be written as follows in terms of their elements.

$A = \{2, 4, 6, 8\}$
$B = \{2, 4, 6, 8\}$

Although the two sets A and B have been described differently, when they are written in terms of their elements, we get the same set. Sets which have the same elements are known as **equal sets**. Accordingly, A and B are equal sets. If two sets A and B are equal then we write $A = B$.

Equivalent sets

If the number of elements in the two sets A and B are equal, that is, if $n(A) = n(B)$, then the sets A and B are known as equivalent sets.

If A and B are equivalent sets, we denote this by $A \sim B$.

Example 2

$X = \{\text{odd numbers between 0 and 10}\}$
$Y = \{\text{vowels in the English alphabet}\}$

By writing the elements of these sets, show that they are equivalent sets.

$x = \{1, 3, 5, 7, 9\}$ $n(X) = 5$
$y = \{a, e, i, o, u\}$ $n(Y) = 5$

Since $n(X) = n(Y)$, X and Y sets are equivalent sets.

Note :- Although all pairs of equal sets are equivalent, all pairs of equivalent sets need not be equal.

Exercise 22.1

1. From the sets given below, select and write the finite sets and the infinite sets separately.

i. $A = \{\text{multiples of 5 from 0 to 50}\}$
ii. $B = \{\text{integers}\}$
iii. $C = \{\text{numbers that can be written using only the digits 0 and 1}\}$
iv. $D = \{\text{digits of the number 25265}\}$
v. $E = \{\text{positive integers which are not prime}\}$

2. Write each of the following sets in terms of their elements and then write all pairs of equal sets and all pairs of equivalent sets.

P = {positive multiples of 3 below 10}
Q = {letters of the word "net"}
R = {odd numbers between 0 and 10}
S = {digits of the number 3693}
T = {vowels in the English alphabet}
v = {letters of the word "ten"}

3. Write 3 examples of finite sets.

4. Write 3 examples of infinite sets.

5. Write three sets which are equivalent to the set {2, 3}.

22.2 The universal set and subsets

Subsets

When two sets A and B are considered, if all the elements in set B are also in set A, then set B is known as a **subset** of set A.

As an example, let us consider the two sets given below which are expressed in terms of their elements.

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6\}$

Since all the elements in set B are in set A, set B is a subset of set A. This is denoted by $B \subset A$ or $A \supset B$, and is read as " B is a subset of A".

Now let us consider another set C. If $C = \{1, 2, 7\}$, then not every element in C belongs to A. Therefore C is not a subset of A. This is denoted by $C \not\subset A$.

Example 1

P = {multiples of 6 between 0 and 20}
Q = {multiples of 3 between 0 and 20}

Write the elements of each of the above sets and select the subset.
$P = \{6, 12, 18\}$

$Q = \{3, 6, 9, 12, 15, 18\}$
As all the elements of P are in Q, P is a subset of Q.

Example 2

Write all the subsets of the set $X = \{1, 2\}$.

It is evident that $\{1\}$ and $\{2\}$ are two subsets. Observe that $\{1, 2\}$ is also a subset. In fact, if two sets A and B are equal, then A is a subset of B and B is a subset of A. Furthermore, the null set is considered to be a subset of every set.

Since the null set and the set itself are subsets of the given set, $\{\ \}$ and $\{1, 2\}$ are subsets of the above set X.

Accordingly, the above set X has 4 subsets which are $\{\ \},\{1\},\{2\},\{1, 2\}$.

Example 3

Write all the subsets of the set $Y = \{3, 5, 7\}$.

$\{\ \}$, $\{3\}$, $\{5\}$, $\{7\}$, $\{3, 5\}$, $\{3, 7\}$, $\{5, 7\}$, $\{3, 5, 7\}$
There are 8 subsets.

Universal sets

In a study conducted on the students of your school, several subsets may come under consideration.

The following can be given as examples.

{Students in grade 9}
{Female students}
{Students sitting for the G.C.E (O/L) examination this year}

The elements of all the above sets are contained in the set of all students of the school. This set can be considered as the **universal set** relevant to the above study.

Let us consider another example.

When we consider the sets of even numbers, odd numbers, triangular numbers and prime numbers, we see that they are all subsets of the set of integers. Therefore the set of integers can be considered as the universal set.

A **universal set** is a set which contains all the elements under consideration. Universal sets are denoted by ε.

As another example, suppose that the numbers 1, 2, 3, 4, 5 and 6 are written on the 6 sides of a cubic die. By rolling this die once, the score that can be obtained is one of the numbers 1, 2, 3, 4, 5 and 6.Therefore, we obtain {1, 2, 3, 4, 5, 6} as the set of possible outcomes. This is the universal set of all possible outcomes that can be obtained when a die is rolled once.

This can be expressed as ε = {1, 2, 3, 4, 5, 6}. A few subsets of this universal set are given below.

A = {odd numbers} A = {1, 3, 5}
B = {values greater than 4} B = {5, 6}
C = {even prime numbers} C = {2}

Example 4

Write a universal set for A; A = {2, 4, 6, 8}

ε = {numbers between 1 and 10}

Exercise 22.2

1. Write 8 subsets of the set A = {2, 5, 8, 10, 13}.

2. Determine whether each of the following statements is true or not.

i. {1, 2, 3 } $\subset$ {numbers divisible by 5}
ii. {4, 9, 16 } $\subset$ {square numbers}
iii. {cylinder} $\subset$ {polygons}
iv. {red} $\subset$ {colours of the rainbow}
v. {solution of $2x - 1 = 7$} $\subset$ {even numbers}

3. Write a universal set for the set A; A = {a, e, i, o, u}

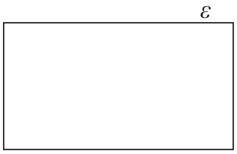
4. For each of the following parts, name a suitable universal set, such that the given sets are subsets.

i. {5, 10, 15, 20, 25} , {10, 100, 100, ...}
ii. {countries with more than 90% literacy},{countries which are not bordered by an ocean}
iii. {January, March, May, August}, {months with 31 days}
{months during which the members of your family celebrate their birthdays}

22.3 Venn diagrams

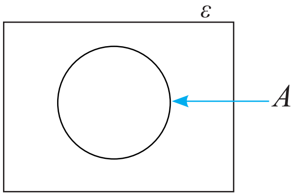
You have learnt how to represent sets in a Venn diagram in earlier grades. In Venn diagrams, sets are represented by closed figures.

The universal set is represented in a Venn diagram by a rectangle as shown below.



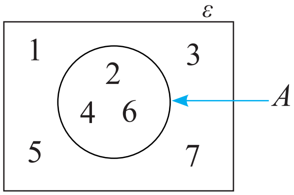
The subsets of a universal set are represented using round or oval shaped figures (circles or ellipses)

We represent a subset A within the universal set as follows.

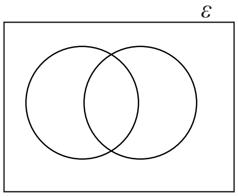


Example 1

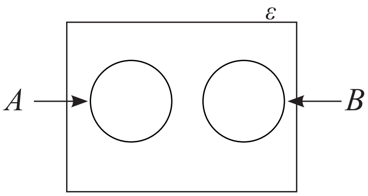
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$
$A = \{2, 4, 6,\}$
Represent the above sets in a Venn diagram.



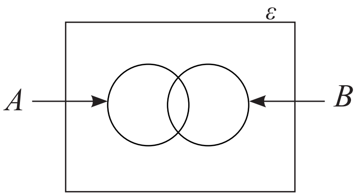
Two subsets of a universal set are generally represented as below.



Special instances of two subsets of a universal set are shown below.

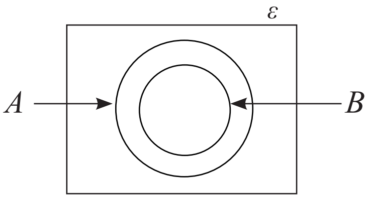


When the two sets *A* and *B* have no elements in common

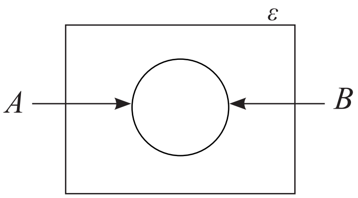


When *A* and *B* have common elements

6



When *B* is a subset of *A*



When *A* and *B* are equal

You will learn more about these four instances and the corresponding regions under the next section on the intersecton of sets, union of sets and disjoint sets.

22.4 Intersection of sets, union of sets and disjoint sets

Intersection of sets

When two or more sets are considered, the set consisting of the elements which are common to all the sets is known as their intersection. When two sets *A* and *B* are considered, their intersection is denoted by $A \cap B$.

As an example, let us consider the pair of sets given below,

$A = \{1, 2, 3, 4, 5, 7\}$

$B = \{2, 5, 6, 7\}$

The set consisting of the elements common to both *A* and *B* is $\{2, 5, 7\}$.

Therefore, the intersection of the sets *A* and *B* is $A \cap B = \{2, 5, 7\}$.

Example 1

M = {students of KannangaraVidyalaya who play cricket}

N = {students of KannagaraVidyalaya who play football}

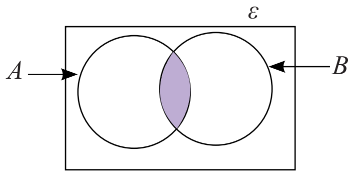
Write the set $M \cap N$ in descriptive form.

$M \cap N$ = {students of KannangaraVidyalaya who play both cricket and football}

Now let us consider how the intersection of two sets is represented in a Venn diagram.

Suppose the two sets *A* and *B* have common elements.

That is, *A* and *B* have a non - empty intersection. There should be a region to represent the set of elements common to both these sets. The Venn diagram depicting this is given below.



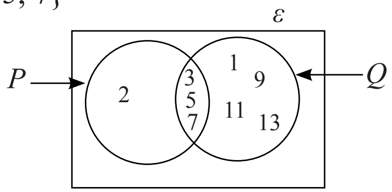
The shaded region in the figure is common to both the sets *A* and *B*. Therefore the elements that are common to these two sets can be included here.

Let us consider how to represent two sets with a non-empty intersection in a Venn diagram, through the following example.

Example 2

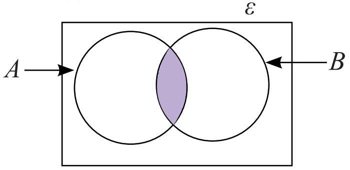
Write the following two sets by listing their elements and then find their intersection. Represent all the elements in a Venn diagram.

P ={prime numbers between 0 and 10}
Q ={odd numbers between 0 and 15}
$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9, 11, 13\}$
$\therefore P \cap Q = \{3, 5, 7\}$

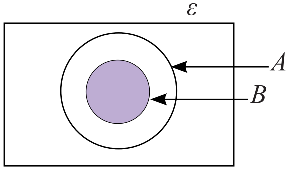


The intersecton of two sets under different conditions are given below.

(i) $A \cap B$

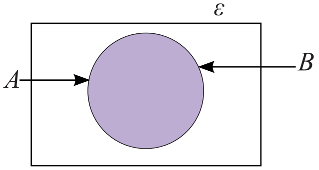


(ii) $A \cap B$ when $B \subset A$



$A \cap B = B$ when $B \subset A$

(iii) $A \cap B$ when $A = B$



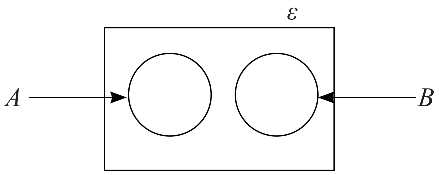
$A \cap B = A = B$ when $A = B$

Let us consider how two sets which do not have common elements represented in a Venn diagram.

Disjoint sets

If two sets have no elements in common, then they are known as disjoint sets. In other words, if two sets A and B are such that $A \cap B = \emptyset$, then A and B are disjoint sets.

Disjoint sets can be represented in a Venn diagram as shown below.

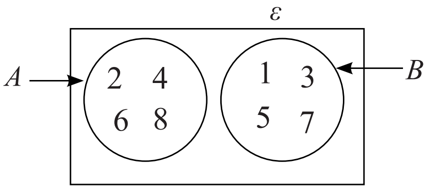


As an example, let us consider the two sets shown below.

$A = \{2, 4, 6, 8\}$

$B = \{1, 3, 5, 7\}$

Since $A \cap B = \emptyset$, A and B are disjoint sets.



$A \cap B = \emptyset$

Union of sets

When two or more sets are considered, the set which consists of all the elements in these sets is known as the **union of these sets**. When two sets A and B are considered, their union is denoted by $A \cup B$.

As an example, let us consider the pair of sets given below.

$A = \{1, 3, 5, 7, 8\}$

$B = \{2, 3, 4, 6, 7, 8\}$

The union of the sets A and B is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Accordingly, the union of the sets A and B is $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Example 1

P = {prime numbers between 0 and 10}
P = {odd numbers between 0 and 10}

Write these sets in terms of their elements and write $P \cup Q$ also in terms of its elements. Furthermore, express the union in terms of a common characteristic of its elements.

$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9\}$

$P \cup Q = \{1, 2, 3, 5, 7, 9\}$

The union expressed in terms of a common characteristic;
$P \cup Q$ = {numbers between 0 and 10 which are either prime or odd}

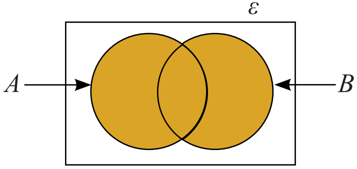
Example 2

X = {students in KannangaraVidyalaya who play cricket}
Y = {students in KannangaraVidyalaya who play football}

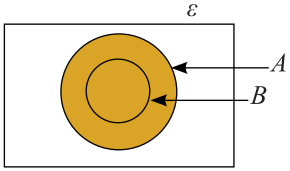
$X \cup Y$ = {students in Kannangara Vidyalaya who play either cricket or football or both}

Now, let us see how to represent the union of sets in a Venn diagram.

(i) $A \cup B$

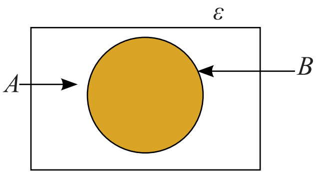


(ii) $A \cup B$ when $B \subset A$



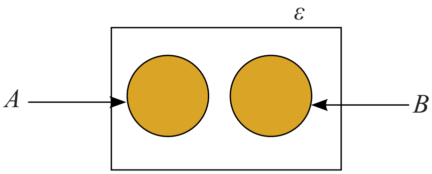
$A \cup B = A$

(iii) $A \cup B$ when $A = B$



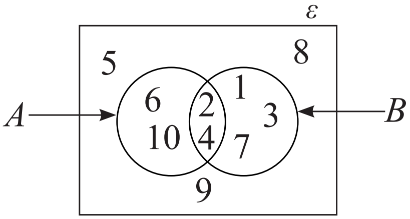
$A \cup B = A = B$ when $A = B$

(iv) $A \cap B = \emptyset$



Example 3

Answer the given questions based on the following Venn diagram.



i. Write set A in terms of its elements.
ii. Write set B in terms of its elements.
iii. Write the universal set ε in terms of its elements.
iv. Express $A \cap B$ in terms of its elements.
v. Express $A \cup B$ in terms of its elements.

i. $A = \{2, 4, 6, 10\}$
ii. $B = \{2, 1, 4, 7, 3\}$
iii. $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
iv. $A \cap B = \{2, 4\}$
v. $A \cup B = \{6, 10, 2, 4, 1, 3, 7\}$

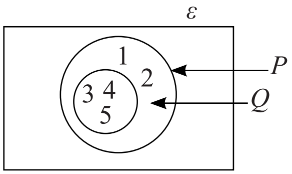
Example 4

$P = \{1, 2, 3, 4, 5\}$
$Q = \{3, 4, 5\}$

i. Represent the above sets in a Venn diagram.
ii. Express $P \cap Q$ and $P \cup Q$ in terms of their elements.

i. $P \cap Q = \{3, 4, 5\}$
$P \cup Q = \{1, 2, 3, 4, 5\}$

ii.



Exercise 22.4

1. The sets P, Q and R are defined as follows.

$P = \{1, 3, 6, 8, 10, 13\}$
$Q = \{1, 6, 7, 8\}$
$R = \{2, 3, 9, 10, 12\}$

Express each of the following sets in terms of its elements.

i. $P \cap Q$ **ii.** $P \cap R$ **iii.** $Q \cap R$ **iv.** $P \cup Q$ **v.** $P \cup R$ **vi.** $Q \cup R$

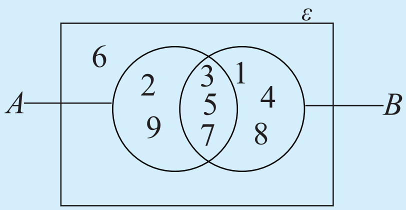
2. The sets A, B and C are defined as follows.

A = {counting numbers from 1 to 12}
B = {prime numbers less than 10}
C = {factors of 12}

- **i.** Write each of the above sets in terms of its elements.
- **ii.** Write each of the following sets in terms of its elements.

i. $A \cap B$ **ii.** $A \cap C$ **iii.** $B \cap C$ **iv.** $A \cup B$ **v.** $A \cup C$ **vi.** $B \cup C$

3. Consider the Venn diagram given below.



Write each of the following sets in terms of its elements.

i. A
ii. B
iii. $A \cup B$
iv. $A \cap B$

22.5 Complement of a set

Let us consider a subset A of a universal set. The set of elements in the universal set which do not belong to the set A is known as the **complement of A**.

Consider the following example.

If we take,
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{2, 4, 6\}$,
then the set consisting of all the elements in the universal set which are not in set A is $\{1, 3, 5, 7\}$.

This set is the complement of the set A. The complement of the set A is denoted by A'. Accordingly,
we can write, $A' = \{1, 3, 5, 7\}$.

Example 1

By considering the given universal set (ε) and its subset B, write the set B' in terms of its elements.

$\varepsilon = \{5, 10, 15, 20, 25, 30, 35\}$
$B = \{10, 20, 30\}$

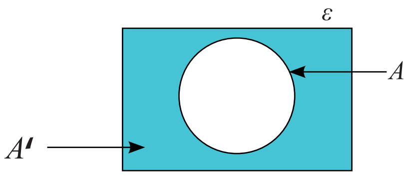
$B' = \{5, 15, 25, 35\}$

Example 2

If $\varepsilon = \{\text{birds}\}$ and
$P = \{\text{birds that make nests}\}$, then write P' in descriptive form.
$P' = \{\text{birds that do not make nests}\}$

Now let us see how to represent the complement of a set in a Venn diagram.

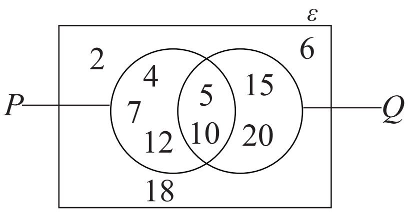
If A is a subset of a universal set, then A' is represented in a Venn diagram as follows.



A' is the set of elements which belong to ε but not to A. Therefore the whole region in the Venn diagram which does not belong to A, belongs to A'.

Example 3

Find the following using the information in the Venn diagram.



i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

i. $P' = \{2, 6, 15, 18, 20\}$
ii. $Q' = \{2, 4, 6, 7, 12, 18\}$
iii. $P \cap Q = \{5, 10\}$
iv. $P \cup Q = \{4, 5, 7, 12, 15, 20, 10\}$

Exercise 22.5

1. ε = {Sakindu, Ravindu, Sanindu, Pavindu, Nithindu}
A = {Sakindu, Pavindu}
B = {Ravindu,Sanindu, Nithindu}
C = {Sakindu, Sanindu, Pavindu}

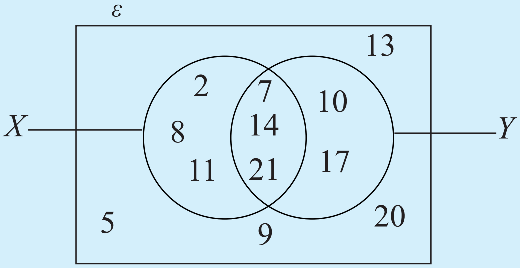
Write each of the following sets in terms of its elements, based on the above given information.

i. A' **ii.** B' **iii.** C'
iv. $A \cap C$ **v.** $A \cap B$ **vi.** $B \cap C$

2. If $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{1, 3, 5, 7, 9, 10\}$ and $Q = \{2, 4, 5, 7, 8\}$, represent ε, P and Q in a Venn diagram and using the Venn diagram, write each of the sets given below in terms of its elements.

i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

3. By considering the Venn diagram given below, write each of the given sets in terms of its elements.



i. X **ii.** Y **iii.** $X\cap Y$

iv. $X\cup Y$ **v.** X' **vi.** Y'

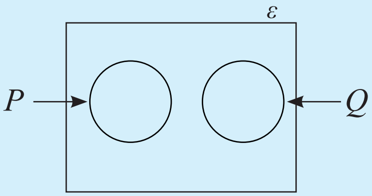
Miscellaneous Exercise

1. Represent the following information in the given Venn diagram.

$\varepsilon=\{1, 2, 3, 4, 5, 6, 7, 8, \}$

$P=\{2, 4, 6\}$

$Q=\{1, 5, 8\}$



Write each of the sets given below in terms of its elements.

a. $P\cap Q$

b. $P\cup Q$

c. P'

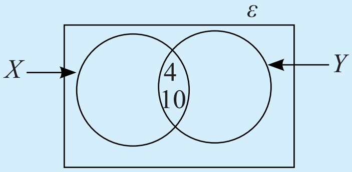
d. Q'

2. Include the elements in the following sets in the given Venn diagram.

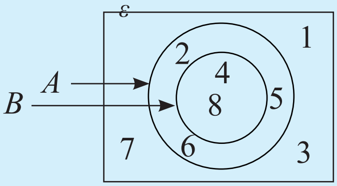
$\varepsilon=\{2, 3, 4, 5, 7, 8, 10, 12 \}$

$P=\{2, 4, 10\}$

$Q=\{3, 4, 8, 10\}$



3. Answer the following based on the information in the Venn diagram.



i. Write set A in terms of its elements.
ii. Write set B in terms of its elements.
iii. Write set ε in terms of its elements.
iv. $A \cap B$ set in terms of its elements.
v. $A \cup B$ set in terms of its elements.
vi. A' set in terms of its elements.

Summary

- Sets with a finite number of elements are called finite sets.
- Sets with an infinite number of elements are called infinite sets.
- Sets with the same elements are called equal sets.
- Sets with the same number of elements are called equivalent sets.
- A sets which contains all the elements under consideration is called universal set.

23

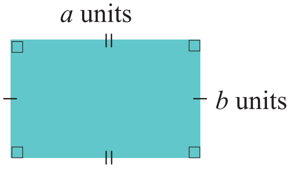
Area

By studying this lesson you will be able to;

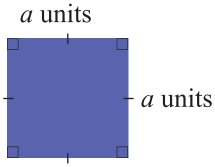
- find the area of a parallelogram,
- find the area of a trapezium,
- find the area of a circle.

Area

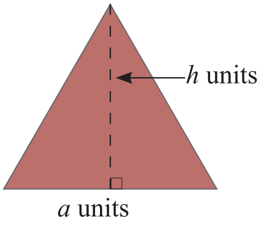
Area can be considered as a quantity that defines the spread of a surface. In grades 7 and 8 you learnt how to find the area of a square lamina, a rectangular lamina and a triangular lamina. Let us recall what was learnt.



If the area of a rectangular lamina of length a units and width b units is A square units, then $A = a \times b$.



If the area of a square lamina of side length a units is A square units, then $A = a^2$.



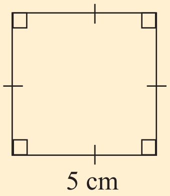
If the area of a triangular lamina of base a units and corresponding perpendicular height h units is A square units, then $A = \frac{1}{2} \times a \times h$.

Do the following review exercise in order to establish these facts further.

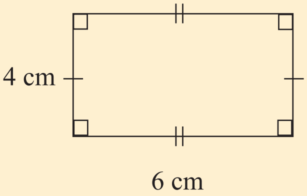
Review Exercise

1. Find the area of each plain figure shown below.

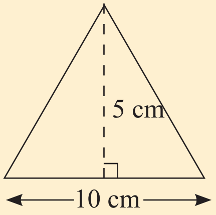
i



ii



iii



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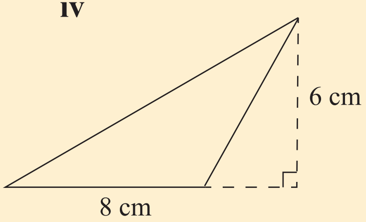
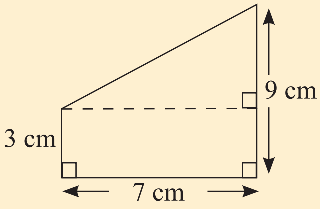
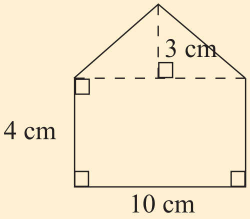


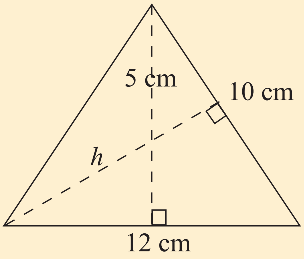
v



vi



2. In the triangle shown in the figure, the perpendicular height corresponding to the base of length 12 cm is 5 cm, and the perpendicular height corresponding to the base of length 10 cm is h cm.



i. Find the area of the triangle.
ii. Find the value of h.

3. (*a*) What is the perimeter of an equilateral triangular lamina of side length 12 cm?

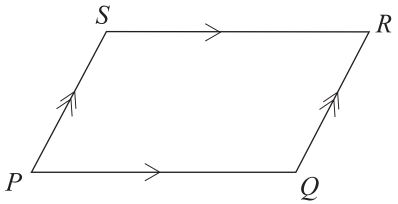
(*b*) Consider a square lamina with the same perimeter as that of the above triangular lamina.

i. Find the side length of the square lamina.
ii. Find the area of the square lamina.

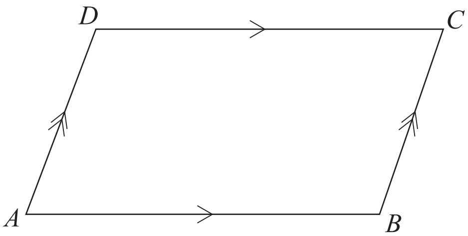
The area of a parallelogram

A quadrilateral with opposite sides parallel to each other is called a parallelogram. We learnt in grade 8 that the opposite sides of a parallelogram are equal. Accordingly, in the parallelogram *PQRS*,

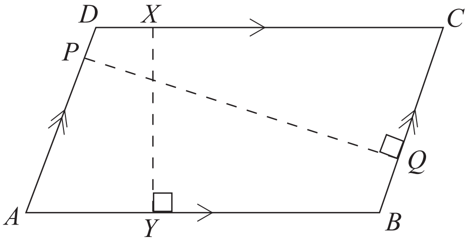
$PQ // SR$ and $PS // QR$.
$PQ = SR$ and $PS = QR$



23.1 The base and height of a parallelogram



Any side of the parallelogram given in the figure can be considered as the base. How the height of the parallelogram corresponding to each base is defined is explained below.

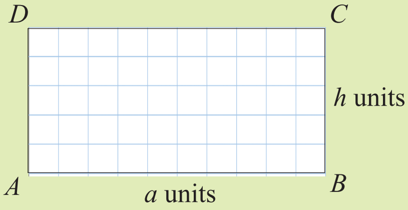


Suppose *AB* is considered as the base of the parallelogram. *AB* and *DC* (the side opposite *AB*) are parallel to each other. According to the figure, the perpendicular distance between these two sides is *XY*. Therefore, *XY* is the perpendicular height corresponding to the base *AB*. When *BC* is considered as the base, the perpendicular distance between the parallel sides *BC* and *AD* according to the figure is *PQ*. Therefore, *PQ* is the perpendicular height corresponding to the base *BC*.

Through the following activity, let us understand how to construct a formula for the area of a parallelogram.

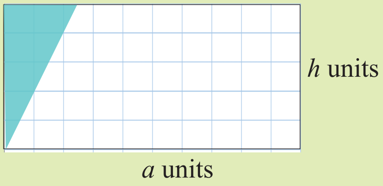
Activity 1

Step 1: In your exercise book, draw a rectangle which is equal in size to that given in the figure. Let us consider its length to be a units and its width to be h units.

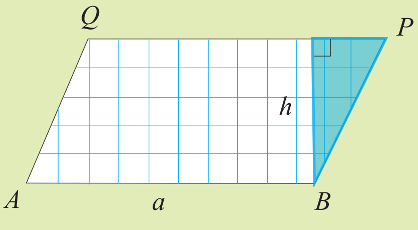


Step 2: Draw another rectangle with the same measurements on another square ruled paper and cut it out.

Step 3: Take that rectangle and cut off the shaded portion as shown below.



Step 4: Create a parallelogram by pasting the triangular portion that was cut off in your exercise book, as shown in the figure. Name it $ABPQ$.



Step 5: Find the area of the rectangle drawn initially, in terms of a and h.

Understand that the area of the parallelogram and the area of the initial rectangle are equal to each other.

Area of parallelogram $ABPQ$ = Area of rectangle $ABCD$
$= a \times h$ square units

Observe that h is the perpendicular height corresponding to the base AB.

According to these facts, a formula for the area of a parallelogram can be given as below.

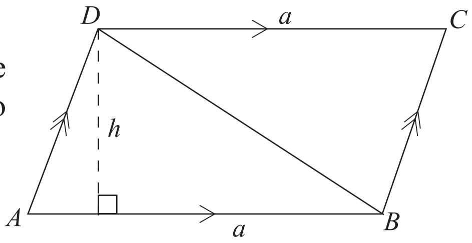
Area of a parallelogram = Length of the base × Perpendicular height corresponding to the base

Let us now consider another method by which the areas of a parallelogram can be found.

The area of a parallelogram can be found by finding the areas of triangles as well.

Consider the parallelogram $ABCD$.

Suppose the length of the base AB is a units and the corresponding height is h units. The parallelogram $ABCD$ is divided into two triangles ABD and BCD by the diagonal DB.



$$\text{The area of triangle } ABD = \frac{1}{2} \times a \times h$$

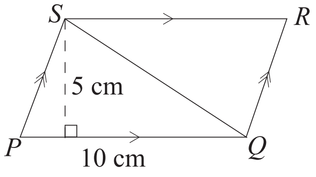
$$\begin{aligned}\text{The area of triangle } BCD &= \frac{1}{2} \times DC \times h \\ &= \frac{1}{2} \times a \times h \text{ (Since } AB = DC)\end{aligned}$$

$$\begin{aligned}\text{Area of parallelogram } ABCD &= \text{Area of triangle } ABD + \text{Area of triangle } BCD \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times a \times h \\ &= \frac{ah}{2} + \frac{ah}{2} = \frac{2ah}{2} \\ &= ah\end{aligned}$$

$\therefore$ The area of the parallelogram *ABCD* is *ah* square units.

Example 1

Find the area of the parallelogram *PQRS*.



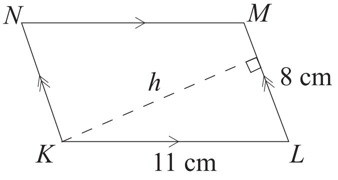
The area of the parallelogram *PQRS* $= 10 \times 5$

$= 50$

Therefore, the area of the parallelogram is 50 cm^2.

Example 2

If the area of the parallelogram *KLMN* is 48 cm^2, find the value of h.



The area of the parallelogram *KLMN* = 48 cm^2

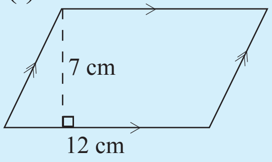
$$\begin{aligned} \text{Therefore, } 8 \times h &= 48 \\ h &= \frac{48}{8} \\ h &= 6 \end{aligned}$$

Therefore, $h = 6$ cm.

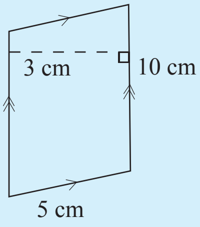
Exercise 23.1

1. Find the area of each parallelogram given below.

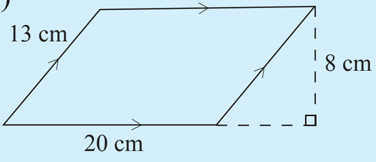
(i)



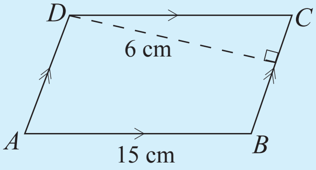
(ii)



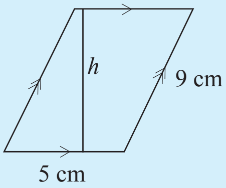
(iii)



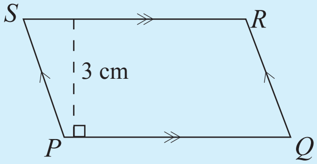
2. If the perimeter of the parallelogram $ABCD$ is 52 cm, find its area.



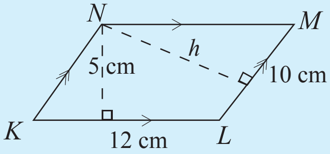
3. If the area of the parallelogram in the figure is 35 cm^2, find the value of h.



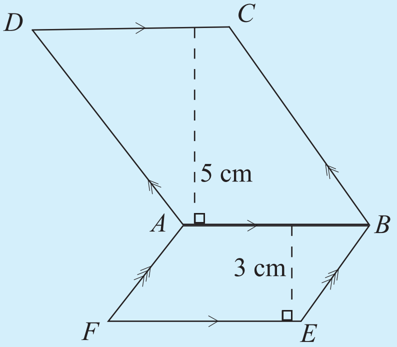
4. If the area of the parallelogram $PQRS$ is 105 cm^2, calculate the length of the side PQ.



5. **i.** Find the area of the parallelogram $KLMN$.
ii. Find the value of h.

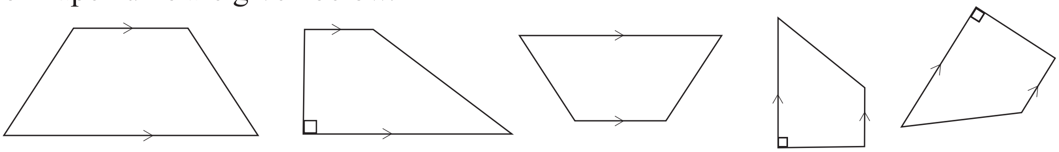


6. If the area of the parallelogram $ABCD$ is 30 cm^2, find the area of the parallelogram $ABEF$.

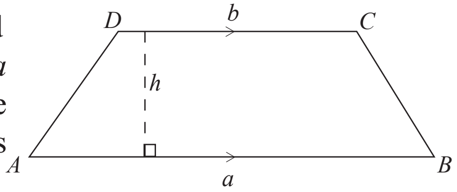


23.2 The area of a trapezium

A quadrilateral with one pair of sides parallel is called a trapezium. Several figures of trapeziums are given below.

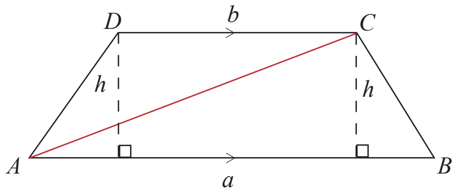


Let us develop a formula for the area of a trapezium.



Let us take the lengths of the parallel sides AB and DC of the trapezium given in the figure as a units and b units respectively and the perpendicular distance between these two sides as h units.

Let us find the area of the trapezium by adding the areas of the two triangles obtained by drawing the diagonal AC of the trapezium.



$$\text{Area of triangle } ABC = \frac{1}{2} \times AB \times h$$

$$\text{Area of triangle } ACD = \frac{1}{2} \times DC \times h$$

$$\text{Area of trapezium } ABCD = \text{Area of triangle } ABC + \text{Area of triangle } ACD$$

$$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$$

$$= \frac{1}{2} \times h \times (AB + DC)$$

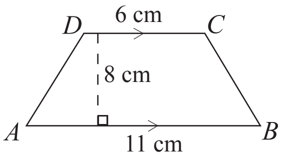
$$= \frac{1}{2} \times (AB + DC) \times h$$

$$= \frac{1}{2} \times (a + b) \times h.$$

The area of a trapezium $= \frac{1}{2} \times \left(\begin{array}{c}\text{The sum of the}\\ \text{lengths of the parallel}\\ \text{sides}\end{array}\right) \times \left(\begin{array}{c}\text{The perpendicular}\\ \text{distance between the}\\ \text{parallel sides}\end{array}\right)$

Example 1

Find the area of the trapezium *ABCD*.

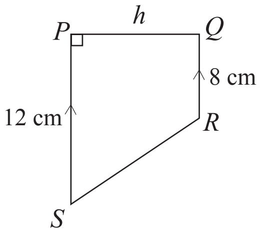


$$\text{The area of the trapezium } ABCD = \frac{1}{2} \times (11 + 6) \times 8$$

$$= \frac{1}{\cancel{2}_1} \times 17 \times \cancel{8}^4$$

$$= \underline{\underline{68 \text{ cm}^2}}$$

Example 2



If the area of the trapezium *PQRS* is 70 cm², find the value of *h*.

$$\text{The area of the trapezium } PQRS = \frac{1}{2} \times (12 + 8) \times h$$

$$= \frac{1}{\cancel{2}_1} \times \overset{10}{\cancel{20}} \times h$$

Since the area is given as 70 cm²,

$$10\,h = 70$$

$$h = \frac{70}{10}$$

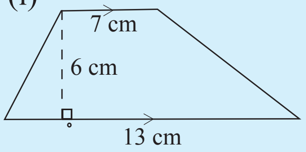
$$h = 7$$

Therefore, $h = 7$ cm.

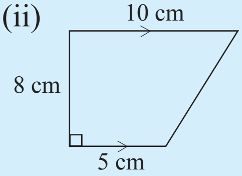
Exercise 23.2

1. Find the area of each trapezium given below.

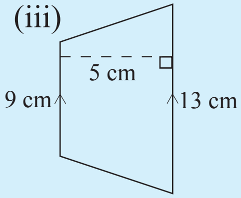
(i)



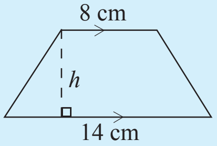
(ii)



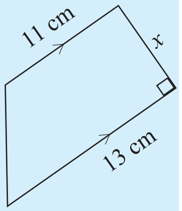
(iii)



2. If the area of the trapezium in the figure is 88 cm^2, find the value of h.

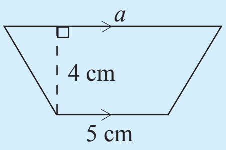


3. If the area of the trapezium in the figure is 60 cm^2, find the value of x.



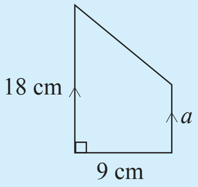
4. Find the length marked as a in each trapezium given below. The area of each trapezium is given below the figure.

i.



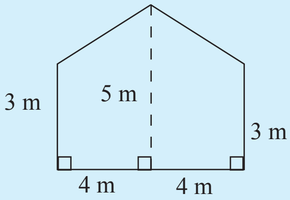
The area is 26 cm^2.

ii.



The area is 135 cm^2.

5. The side view of a wall is given in the figure. Find the area of the wall according to the given measurements.



6. The area of a trapezium is 30 cm^2. The perpendicular distance between the parallel sides is 3 cm.

i. Give three pairs of integral values that the lengths of the parallel sides can take.

ii. Give three pairs of non – integral values that the lengths of the parallel sides can take.

23.3 The area of a circle

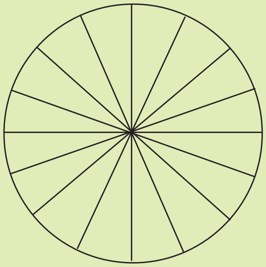
We have learnt how to find the area of a lamina that takes the shape of a rectangle, a square, a triangle, a parallelogram or a trapezium.
Now let us consider how the area of a circular shaped lamina can be found.

To do this, let us first engage in the activity given below.

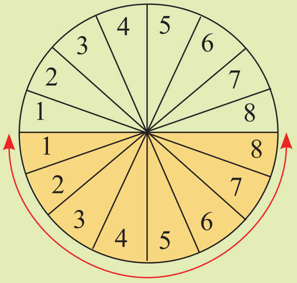
Activity 2

Step 1 : Draw a circle of radius 6 cm on a sheet of paper.

Step 2 : Divide the circle into the maximum possible number of sectors (about 16) by drawing straight lines through the center.



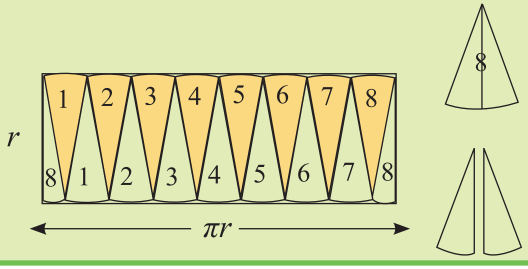
Step 3 : Colour half the circle and number all the sectors consecutively as shown in the figure.



$$\frac{2\pi r}{2} = \pi r$$

Step 4 : Separate out all the sectors by cutting along the drawn lines.

Step 5 : Paste the separated sectors such that a rectangular shape (approximately) is obtained as shown in the figure. (Understand that the accuracy increases as the number of sectors increases.)



Since paper is not wasted, the areas of the circle and the rectangle should be equal. Find the area of the rectangle as shown below by considering the radius of the circle as r.

The length of the rectangle that is obtained = circumference of the circle $\times \frac{1}{2}$

$$= 2\pi r \times \frac{1}{2}$$

$$= \pi r$$

The width of the rectangle that is obtained = r

The area of the rectangle = length × width

$$= \pi r \times r$$

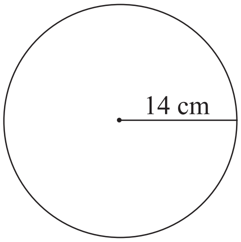
$$= \pi r^2$$

$\therefore$ Therefore, the area of a circle of radius $r = \pi r^2$

In calculations, we use 3.142 or $\frac{22}{7}$ for the value of π.

Example 1

Find the area of a circular lamina of radius 14 cm.



The area of the circular lamina = πr^2

$$= \frac{22}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \times 14$$

$$= 616$$

$\therefore$ Therefore, the area of the circular lamina is 616 cm^2.

Example 2

Calculate the radius of a circular lamina of area 154 cm^2.

The area of the circular lamina $= \pi r^2$

$$= \frac{22}{7} \times r^2$$

The area of the circular lamina is given as 154 cm^2.

$$\frac{22}{7} r^2 = 154$$

Therefore, $\frac{22}{\cancel{7}} r^2 \times \cancel{7} = 154 \times 7$

$$\frac{\cancel{22} r^2}{\cancel{22}} = \frac{1078}{22} = 49$$

$$r^2 = 49$$

Therefore, $r = 7$ or $r = -7$.

However, the radius cannot be a negative value.
Therefore, the radius of the circle is 7 cm.

Exercise 23.3

1. The following are the dimensions of some circular laminas. Find the area of each lamina (Use $\frac{22}{7}$ for the value of π).

i. Radius 14 cm **ii.** Radius 21 cm **iii.** Diameter 7 cm **iv.** Diameter 21 cm

2. The following are areas of some circular laminas. Calculate the radius of each lamina.

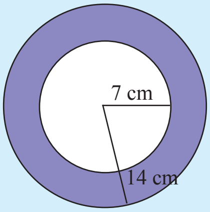
i. 616 cm^2 **ii.** 1386 cm^2 **iii.** $38\frac{1}{2}$ cm

3. Consider the largest circular lamina that can be cut out from a square lamina of area 196 cm^2.

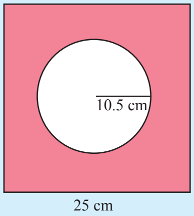
i. What is the radius of this circular lamina?
ii. What is the area of the circular lamina?

4. Find the area of the shaded part in each figure given below.

i.



ii.



5. What is the maximum number of circular laminas of radius 7 cm that can be cut out from a rectangular lamina of length 70 cm and width 14 cm?

Summary

- The area of a parallelogram of base length a and height h is ah.
- The area of a trapezium of which the lengths of the two parallel sides are a and b and the perpendicular distance between the two parallel sides is h is $\frac{1}{2}(a+b)h$.
- The area of a circle of radius r is πr^2.

24

Probability

By studying this lesson you will be able to;

- identify random experiments,
- write the sample space of a random experiment,
- identify the equally likely outcomes of a random experiment,
- find the probability of an event in a sample space when the outcomes are equally likely.

24.1 Random experiment

Let us consider the experiment of an ordinary coin being tossed. When a coin is tossed, we know that the outcome will be either "head turns up" or "tail turns up". That is, we know all the possible outcomes before the experiment is conducted. However, we cannot say with certainly whether head will turn up or tail will turn up. Furthermore, this experiment can be repeated any number of times under the same conditions. Another feature is that we will not be able to identify a pattern in the outcomes when the experiment is repeated. Experiments with the above features are called **random experiments**.

Random experiments have the following common characteristics.

- The experiment can be repeated any number of times under the same conditions.
- All the possible outcomes of the experiment are known before the experiment is carried out.
- The outcome of the experiment cannot be stated with certainty before the experiment is carried out.
- When the experiment is repeated, a pattern cannot be recognized in the outcomes.

Let us consider another example.

Even though all the outcomes of the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up are known, it is not possible before carrying out the experiment to state with certainty which outcome will occur. Moreover, this experiment can be repeated any number of times under the same conditions, but a pattern in the outcomes cannot be expected. Therefore, rolling an unbiased cubic die and observing the outcome is a random experiment.

Exercise 24.1

1. For each of the following experiments, in the column to the right, mark "✓" if it is a random experiment and "×" if it is not a random experiment.

Experiment	Random/not random
1. Rolling an unbiased tetrahedral die with its faces numbered from 1 to 4, and recording the number on the face which touches the table.	
2. Drawing a bead from a bag which contains beads of one color and recording its colour.	
3. Throwing a ball at a target and observing whether it hits the target or not.	
4. Planting 5 radish seeds and recording the number of seeds that germinate in 5 days.	
5. Checking whether a door opens when a key picked at random from a bunch of three keys is used.	
6. Tossing a ball in the air and observing whether it falls to the ground.	
7. Drawing out two cards from a box containing three cards, each with one of the numbers 1, 3 and 5 written on it, and observing whether the sum of the two numbers on the two cards that are drawn is an odd number.	

24.2 Sample Space

All the possible outcomes of a random experiment can be written as a set. This set which consists of all the possible outcomes of a random experiment is called its sample space. It is usually denoted by S.

For example,

in the experiment of tossing a coin and observing the side that turns up, the set of all possible outcomes, that is, the sample space is $S = \{\text{Head, Tail}\}$. Here $N(S) = 2$.

Similarly, the sample space of the experiment of observing the number which turns up when an unbiased cubic die with its faces numbered from 1 to 6 is rolled is

$$S = \{1, 2, 3, 4, 5, 6\}$$

Here $N(S) = 6$.

Example 1

Write the sample space for the experiment of rolling an unbiased tetrahedral die with its faces numbered from 1 to 4 and recording the number on the face which touches the table.

$$S = \{1, 2, 3, 4\}$$

$$n(S) = 4$$

Example 2

Write the sample space for the experiment of drawing a bead from a bag which contains two black beads and three white beads marked B_1, B_2, W_1, W_2, W_3 respectively, which are identical in all other aspects. What is the value of $n(S)$?

$$S = \{ B_1, B_2, W_1, W_2, W_3 \}$$

$$n(S) = 5$$

Example 3

There are two cards such that R is written on one side and Y is written on the other side. The cards are tossed simultaniously and the letters turned up is recorded. Write the sample space of this experiment.

Getting R on both cards is denoted by (R, R) and getting R on one card and Y on the other is denoted as (R, Y) etc.. Accordingly,

$$S = \{(RR), (RY), (YR), (YY)\}$$

Note: An event is a subset of the sample space of a random experiment.

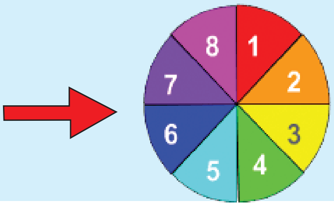
Exercise 24.2

1. Write the sample space of each of the following experiments.

i. Randomly drawing a pen from a bag which contains one pen each of the colors blue, red, black and green and recording the colour. (Assume that the pens are identical in all aspects except the colour)

ii. Recording the number on the card that is drawn at random from a bag containing eleven identical cards numbered 5 to 15.

iii. Recording the number the arrow points to, when the disk shown in the figure is spun and allowed to a stop freely.



iv. A bag contains 4 milk flavoured toffees and 3 orange flavoured toffees of the same size and shape. Randomly drawing a toffee and recording its flavour.

v. Recording the sides that turn up when a coin is tossed twice.

24.3 Equally likely outcomes

When the sample space of a random experiment is considered, if each outcome is equally likely to occur, then that experiment is called an experiment with equally likely outcomes. The outcomes of such an experiment are called equally likely outcomes.

Let us consider a cubic die with its faces numbered 1, 2, 3, 4, 5 and 6. Let us assume that the material it is made of is uniformly distributed throughout the die. Then it is clear that due to symmetry, each face of the die has an equal chance of turning up when the die is rolled. Similarly for a coin. Objects such as these which are symmetrical and are made of a material which is uniformly distributed are called unbiased or fair objects. Experiments such as these, of tossing a fair coin or rolling an unbiased die are considered as important examples when it comes to explaining the theory of probability.

Consider the experiment of rolling a cuboidal die with its faces marked, 1, 2, 3, 4, 5 and 6 and recording the number on the face that turns up. Here, the likelihood of the different sides turning up may not be the same. Therefore such a die is not considered to be a fair die. In such experiments, the outcomes are not equally likely.

Now let us consider another experiment.

It is clear that in the experiment of an unbiased cubic die with four faces painted red and two faces painted blue being rolled and the colour of the face that turns up being recorded, the chance of a red face turning up is greater than a blue face turning up. Therefore, the outcomes of this experiments are not equally likely.

Exercise 24.3

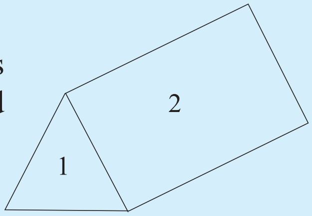
1. For each of the following experiments, determine whether the outcomes are equally likely or not.

i. The four faces of an unbiased tetrahedral die are painted in four different colours, namely, red, blue, yellow and green. Recording the colour of the face which turns up when it is rolled.

ii. Recording the side which turns up when a fair coin is tossed.

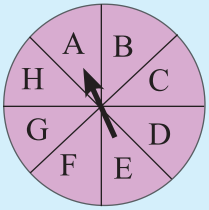
iii. Recording the number on the card which is drawn at random from 10 identical cards which are numbered 1, 1, 1, 1, 2, 2, 2, 3, 3 and 4.

iv. Recording the number on the side which touches the ground when a prism as shown in the figure, with its sides marked with the numbers 1, 2, 3, 4 and 5 is rolled once.



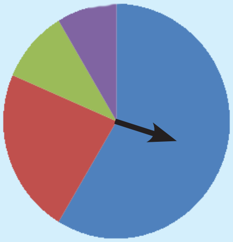
v. Recording the colour of the card drawn randomly from a bag which contains 3 red cards and 4 blue cards which are identical in all other aspects.

vi.



Recording the letter to which the indicator (arrow) which is fixed at the center of a circular disk points, when the disc which has been divided into 8 equal sectors and named A, B, C, D, E, F, G and H as shown in the figure, is spun and allowed to stop freely.

vii.



Recording the color on which the indicator (arrow) which is fixed to the centre of the disc falls when it is spun and allowed to stop freely. Here the disc is divided into unequal sectors and shaded with different colours, and placed on a horizon table top.

24.4 Probability of an event when the outcomes are equally likely

You have learnt that the probability of an outcome of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of a selected outcome} = \frac{1}{\text{total number of outcomes in the sample space of the random experiment}}$$

Consider the experiment of rolling a fair die. Here the probability of a selected outcome is $\frac{1}{6}$ For example, the probability of getting 3 is $\frac{1}{6}$.

Now consider the event of getting an even number. Its probability can be calculated as follows. Since there are three even numbers and three odd numbers, and the outcomes of this experiment are equally likely, the probability of getting an even number is $\frac{3}{6}$.

The probability of an event in the sample space of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of the event} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}$$

This can be written using symbols as follows.

If the number of elements in the sample space S is $n\,(S)$, the number of elements in the event A is $n(A)$ and the probability of event A occurring is $p(A)$, then

$$p\,(A) = \frac{n\,(A)}{n\,(S)}$$

Now let us learn more by considering some examples.

Example 1

Consider the experiment of observing the side that turns up when an unbiased coin is tossed once.

i. Write the sample space of this experiment and find $n(S)$.
ii. If the event A is "head" turns up, write the elements in A and find $n(A)$.
iii. Find $p(A)$, the probability that head turns up.

i. S = {head, tail}
$n\,(S) = 2$

ii. A = {head}
$n\,(A) = 1$

iii. $p\,(A) = \frac{n\,(A)}{n\,(S)}$

$p\,(A) = \frac{1}{2}$

Example 2

Consider the experiment of recording the number on the face that touches the table when an unbiased tetrahedral die with its faces numbered 1, 2, 3 and 4 is rolled.

i. Find the probability of getting 2.
ii. Find the probability of getting an even number.
iii. Find the probability of getting a number greater than 1.

Since the sample space is $S = \{1, 2, 3, 4\}$, $n(S) = 4$.

i. Probability of getting 2 $= \dfrac{1}{4}$

ii. If B is the event of getting an even number,

since $B = \{2, 4\}$, $n(B) = 2$.

$$\therefore p(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

iii. There are 3 numbers greater than 1. (2, 3, 4)

$\therefore$ the probability of getting a number greater than 1 $= \dfrac{3}{4}$

Exercise 24.4

1. Consider the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up.

i. Write the sample space S of all the possible outcomes of this experiment.

ii. Find the value of $n(S)$.

iii. If A is the event of an even number turning up, write the elements of A and find $n(A)$.

iv. Find $P(A)$, the probability of A occurring.

v. Find the probability of a prime number turning up.

2. Consider the experiment of drawing a card at random from a bag containing 8 identical cards marked with the letters A, B, C, D, E, F, G and H and recording the letter on it.

i. Write the sample space.

ii. Find the probability of drawing the card with the letter B marked on it.

iii. Find the probability of drawing a card with a vowel marked on it.

iv. Find the probability of drawing a card with the letter K marked on it.

3. There are 25 identical cards numbered from 1 to 25 in a box. Consider the experiment of drawing a card at random from the box and recording the number on it.

- **i.** Find the probability of drawing the card with 8 marked on it.
- **ii.** Find the probability of drawing a card with a number which is a multiple of 5 marked on it.
- **iii.** Find the probability of drawing a card with an odd number marked on it.
- **iv.** Find the probability of drawing a card with a square number marked on it.

4.

Consider the experiment of spinning the disc in the figure and recording the colour of the sector in which the arrow head lands when the disc stops spinning.

- **i.** Find the probability of the arrow head landing in the blue sector.
- **ii.** Find the probability of the arrow head landing in the red sector.
- **iii.** Find the probability of the arrow head landing in the white sector.

5. In a multiple choice question paper, of the 5 answers that are given for a question, only one is correct. A person picks one of the answers randomly for a question to which he does not know the answer. What is the probability of that answer

- **i.** being correct.
- **ii.** being incorrect.

6. In a bag, there are 3 red beads, 2 black beads and 5 white beads which are identical in all other aspects. Consider the experiment of randomly drawing a bead from the bag and recording its colour.

- **i.** Find the probability of drawing a red bead.
- **ii.** Find the probability of drawing a blue bead.
- **iii.** Find the probability of drawing either a red bead or a white bead.
- **iv.** Find the probability of drawing a black bead.

7. Consider the experiment of recording the day of the week on which a student picked at random was born.

i. Find the probability of the student being a person who was born on a Monday.

ii. Find the probability of the student being a person who was born on a Sunday.

iii. Find the probability of the student being a person who was born on either a Wednesday or a Friday.

iv. Find the probability of the student being a person who was born on a day which is neither a Saturday nor a Sunday.

Summary

In a random experiment with equally likely outcomes,

- $$\text{the probability of a selected outcome} = \frac{1}{\text{total number of outcomes in the sample space of the random experiment}}$$
- $$\text{the probability of the event} = \frac{\text{number of elements in the event}}{\text{number of elements in the sample space}}$$
- $$p\,(A) = \frac{n\,(A)}{n\,(S)}$$

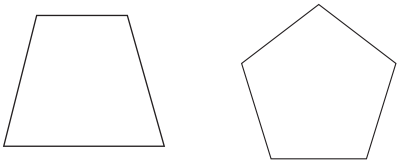
25

Angles of a Polygon

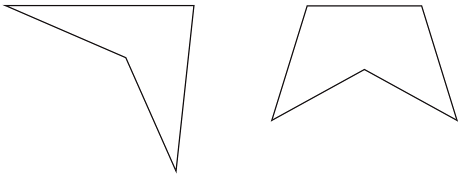
By studying this lesson you will be able to;

- solve geometrical problems related to the interior angles of a polygon,
- solve geometrical problems related to the exterior angles of a polygon,
- solve problems related to regular polygons.

A plane figure bounded by three or more straight line segments is known as a polygon. The two main types of polygons are convex polygons and concave polygons.

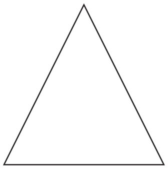


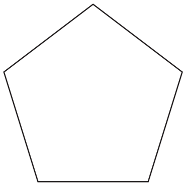
Convex Polygons

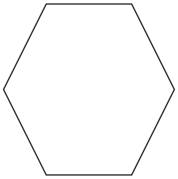


Concave Polygons

Some polygons have special names by which they are identified, which depend on the number of sides they have. Accordingly, polygons which have 3, 4, 5 and 6 sides respectively are known as triangles, quadrilaterals, pentagons and hexagons.







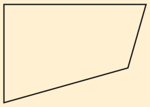
In previous grades you have learnt the following results on the sum of the interior angles of a triangle and of a quadrilateral.

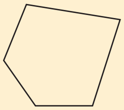
The sum of the interior angles of a triangle is 180°.
The sum of the interior angles of a quadrilateral is 360°.

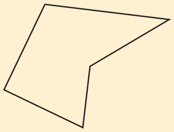
Do the following review exercise to further establish the above facts learnt on polygons in previous grades.

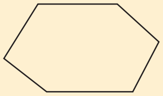
Review Exercise

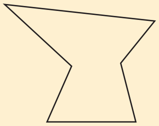
1. From the given figures, select all the convex polygons.


(i)


(ii)


(iii)


(iv)

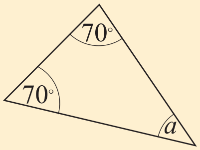

(v)

2. From the statements given below, select the true statements.

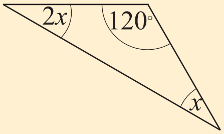
- **a.** A polygon with 7 sides is known as a heptagon.
- **b.** In any polygon, the number of interior angles is equal to the number of sides.
- **c.** A polygon with all sides equal is known as a regular polygon.
- **d.** At each vertex of a polygon, the sum of the interior angle and the exterior angle is 180°.
- **e.** There are 11 interior angles in a decagon.
- **f.** The sum of the exterior angles of a quadrilateral is 180°.

3. Find the magnitude of each of the angles denoted by a lower case letter in each of the following figures.

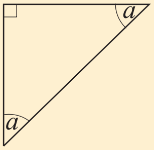
i.



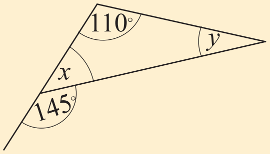
ii.



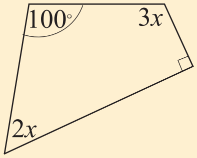
iii.



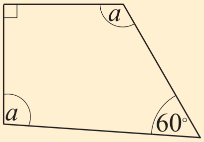
iv.



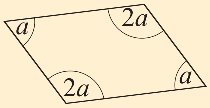
v.



vi.

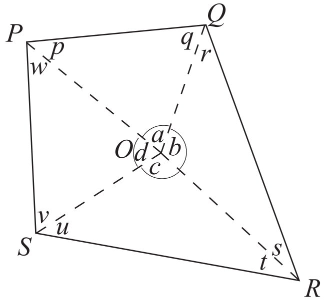


vii.



25.1 The sum of the interior angles of a polygon

Let us first consider how to find the sum of the interior angles of a quadrilateral.



In the given figure, O is any point within the quadrilateral $PQRS$. By joining PO, QO, RO and SO we obtain 4 triangles.

Since the sum of the interior angles of a triangle is 180°,

considering the triangle PQO we obtain, $p + q + a = 180^{\circ}$,

considering the triangle QRO we obtain, $r + s + b = 180^{\circ}$,

considering the triangle RSO we obtain, $t + u + c = 180^{\circ}$,

considering the triangle SPO we obtain, $v + w + d = 180^{\circ}$,

By adding these 4 equations we obtain,

$$(p + q + a) + (r + s + b) + (t + u + c) + (v + w + d) = 180^{\circ} \times 4$$

$$\therefore\ (p + q + r + s + t + u + v + w) + (a + b + c + d) = 720^{\circ}$$

Since a, b, c and d are angles around the point O, $a + b + c + d = 360^{\circ}$.

$$\therefore\ p + q + r + s + t + u + v + w = 720^{\circ} - 360^{\circ}$$

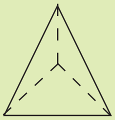
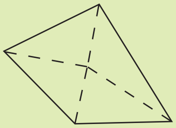
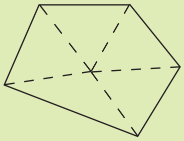
$$= 360^{\circ}$$

$\therefore$ the sum of the interior angles of a quadrilateral is 360°.

Now, to obtain an expression in terms of n for the sum of the interior angles of a polygon which has n sides, let us engage in the following activity.

Activity 1

Copy the following table and complete it.

Polygon	**Figure**	**Number of triangles**	**Sum of the interior angles**
Triangle		3	$180^\circ \times 3 - 360^\circ = 180^\circ$
Quadrilateral		4	$180^\circ \times 4 - 360^\circ = 360^\circ$
Pentagon		5	$180^\circ \times ... - 360^\circ = 540^\circ$
Hexagon		....	....
Heptagon		....	....
Octagon		....	....
Polygon with n sides		....	....

In the above activity, you must have obtained that when the number of sides is n, the sum of the interior angles of the polygon is $180^\circ \times n - 360^\circ$.

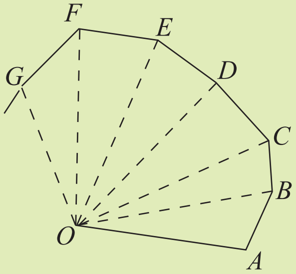
Let us write the expression $180^\circ \times n - 360^\circ$ as follows so that it is easier to remember.

$$\begin{aligned} 180^\circ \times n - 360^\circ &= 90^\circ \times 2n - 90^\circ \times 4 \\ &= 90^\circ (2n - 4) \\ &= (2n - 4) \text{ right angles.} \end{aligned}$$

$\therefore$ The sum of the interior angles of a polygon of n sides $= (2n - 4)$ right angles.

Activity 2

2. Let us find a formula for the sum of the interior angles of a polygon in another way.



Polygon	Number of sides	Name of the polygon	Number of triangles	Sum of the interior angles
OAB	3	Triangle	1	$180^{o} \times 1 = 180^{o}$
OABC	4	Quadrilateral	2	$180^{o} \times$ .... = ...
OABCD	....	....	....	....
OABCDE	....	....	....	....
OABCDEF	....	....	....	....
OABCDEFG	....	....	....	....

i. Consider the above table and find in terms of n, the number of triangles that are formed by joining one vertex of an n-sided polygon to the other vertices of the polygon.

ii. Show that the sum of the interior angles of a polygon of n sides is $180^{o}(n-2)$.

Note: Historically, mathematicians such as the Greek mathematician Euclid, expressed the magnitudes of angles in terms of right angles. For example, the magnitude of the angle on a straight line was said to be 2 right angles and the sum of the interior angles of a quadrilateral was said to be 4 right angles. Accordingly, we can say that the sum of the interior angles of a polygon of n sides is $2n-4$ right angles. However, since we use degrees to measure angles and are familiar with the fact that a right angle is 90^{o}, the sum of the interior angles of a polygon can be remembered as either $90^{o}(2n-4)$ or $180^{o}(n-2)$ or any other equivalent expression which is easy to recall.

Example 1

Find the sum of the interior angles of a nonagon.

$$\text{Sum of the interior angles of a polygon of } n \text{ sides} = 180^\circ\,(n-2)$$

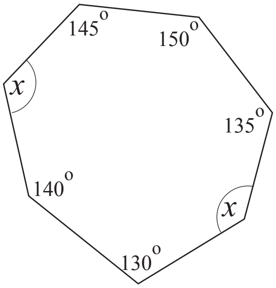
$$\therefore \text{ the sum of the interior angles of a polygon of 9 sides} = 180^\circ\,(9-2)$$

$$= 180^\circ \times 7$$

$$= \underline{\underline{1260^\circ}}$$

Example 2

Find the value of x based on the information in the figure.



Number of sides in the polygon $= 7$

$$\therefore \quad \text{sum of the interior angles} = 180^\circ\,(7-2)$$

$$= 180^\circ \times 5$$

$$= 900^\circ$$

$$\therefore\ 145^\circ + 150^\circ + 135^\circ + x^\circ + 130^\circ + 140^\circ + x = 900^\circ$$

$$700^\circ + 2x = 900^\circ$$

$$2x = 900^\circ - 700^\circ$$

$$2x = 200^\circ$$

$$x = \frac{200^\circ}{2} = 100^\circ$$

Example 3

The sum of the interior angles of a polygon is 1440°. Find the number of sides it has.

If the number of sides is n, the sum of the interior angles $= 180^\circ\,(n-2)$

$$\therefore\ 180^\circ\,(n-2) = 1440^\circ$$

$$n-2 = \frac{1440^\circ}{180} = 8$$

$$n-2 = 8$$

$$n = 10$$

$\therefore$ the number of sides $= 10$.

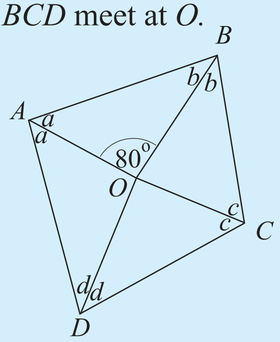
Exercise 25.1

1. Find the sum of the interior angles of each of the polygons given below.

i. Pentagon **ii.** Octagon **iii.** Dodecagon **iv.** Polygon with 15 sides

2. Four of the interior angles of a heptagon are 100°, 112°, 130° and 150°. The remaining angles are equal. Find their magnitude.

3. The bisectors of the interior angles of the quadrilateral $ABCD$ meet at O.

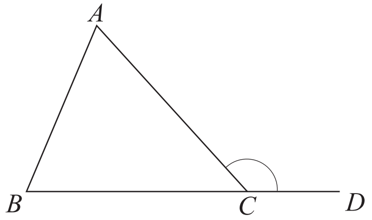


i. Find the value of $a + b + c + d$.
ii. Find the value of $a + b$.
iii. Find the value of $c + d$.
iv. Find the value of $C\hat{O}D$.

4. i. If the sum of the interior angles of a polygon is 1620°, find the number of sides it has.
ii. If the sum of the interior angles of a polygon is 3600°, find the number of sides it has.

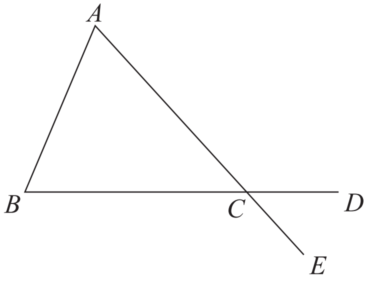
25.2 Sum of the exterior angles of a polygon

First, let us find the sum of the exterior angles of a triangle.



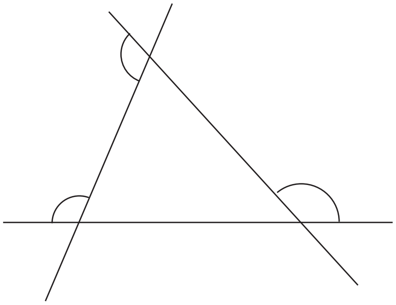
The side BC of the triangle ABC has been produced and point D has been marked on the produced line. The angle $A\hat{C}D$, with the straight line segment CD and the side AC as arms is an exterior angle of this triangle.

As indicated in the figure given below, by producing the side AC too we obtain an exterior angle.

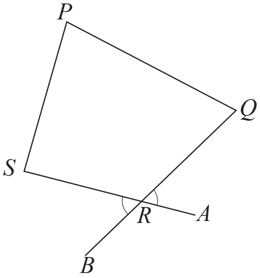


Since vertically opposite angles are equal, this exterior angle is equal in magnitude to the exterior angle $A\hat{C}D$. Either of these two angles can be considered as the exterior angle drawn at the vertex C of the triangle. However, $D\hat{C}E$ is not considered as an exterior angle.

As done above, exterior angles can be drawn at the vertices A and B of the triangle too.



We can define the exterior angles of a quadrilateral similarly.



By producing the side SR of the quadrilateral $PQRS$ up to A we obtain the exterior angle $Q\hat{R}A$ and by producing the side QR up to B we obtain the exterior angle $S\hat{R}B$. Since vertically opposite angles are equal, these two exterior angles are equal.

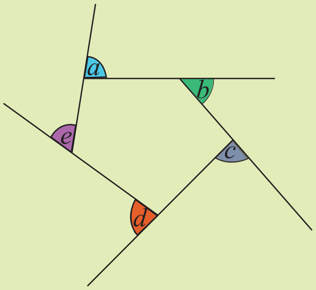
Furthermore, $A\hat{R}B$ is not considered as an exterior angle.

We can define the exterior angles of any polygon similarly.

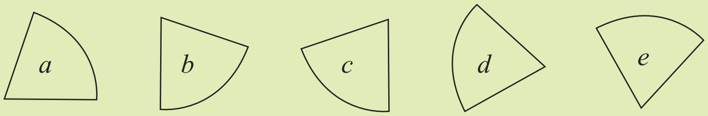
Now, through the following activity, let us determine a value for the sum of the exterior angles of a polygon.

Activity 3

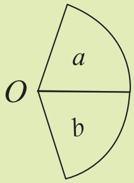
Step 1: Draw a pentagon on a half sheet and name its exterior angles.



Step 2: Using a blade, cut out the exterior angles as laminas (in the form of sectors of circles) and separate them. (By using the same radius when drawing the sectors, the final outcome will be neat)



Step 3: On a sheet of paper, paste the laminas which were cut out, such that their vertices meet at one point and such that they don't overlap each other.



Step 4: Carry out the above steps for a hexagon and a heptagon too.

Step 5: Write the common characteristic of the figures obtained by pasting the exterior angles of the polygons and write what can be concluded through this activity too.

You may have observed in the above activity that for each polygon, the exterior angles cover the angle around a point. Accordingly, it can be concluded that the sum of the exterior angles of a polygon is equal to the sum of the angles around a point. As the sum of the angles around a point is 360°, the sum of the exterior angles of the above polygons is also 360°.

Now let us obtain an expression for the sum of the exterior angles of a polygon which has n sides.
We know that the number of exterior angles and the number of interior angles of a polygon with n sides is n.

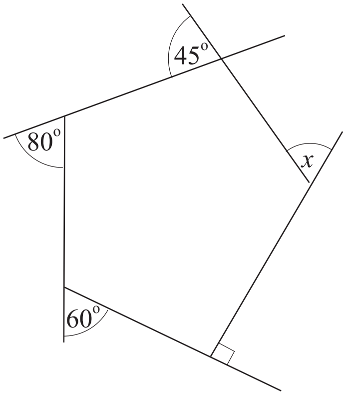
At any vertex of a polygon,
the interior angle + the exterior angle $= 180^\circ$.
$\therefore$ sum of n interior angles + sum of n exterior angles $= 180^\circ \times n$.
But the sum of n interior angles = $(2n - 4)$ right angles = $180^\circ(n - 2)$. Therefore,
$180^\circ(n - 2)$ + sum of n exterior angles $= 180^\circ n$

$$
\begin{aligned}
\therefore \text{ sum of } n \text{ exterior angles} &= 180^\circ n - 180^\circ(n - 2) \\
&= 180^\circ n - 180^\circ n + 360^\circ \\
&= 360^\circ
\end{aligned}
$$

Sum of the exterior angles of a polygon = 360°

Example 1

Find the magnitude of the exterior angle indicated by x, of the given pentagon.

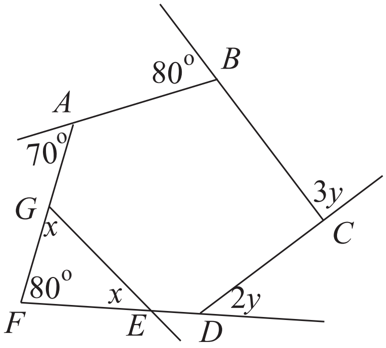


$$
\begin{aligned}
\text{Sum of the exterior angles} &= 360^\circ \\
\therefore\ x + 45^\circ + 80^\circ + 60^\circ + 90^\circ &= 360^\circ \\
x + 275^\circ &= 360^\circ \\
x &= 360^\circ - 275^\circ \\
x &= \underline{\underline{85^\circ}}
\end{aligned}
$$

Example 2

According to the information marked in the figure,

i. find the value of x
ii. find the value of y.



i. Sum of the interior angles of the triangle $EFG = 180^\circ$

$$\therefore\ 80^\circ + x + x = 180^\circ$$
$$2x = 180^\circ - 80^\circ = 100^\circ$$
$$x = \frac{100^\circ}{2}$$
$$x = \underline{\underline{50^\circ}}$$

ii. Sum of the exterior angles of the hexagon $ABCDEG = 360^\circ$

$$\therefore\ 70^\circ + 80^\circ + 3y + 2y + x + x = 360^\circ$$
$$70^\circ + 80^\circ + 5y + 50^\circ + 50^\circ = 360^\circ$$
$$5y = 360^\circ - 250^\circ$$
$$y = \frac{110^\circ}{5}$$
$$y = \underline{\underline{22^\circ}}$$

Example 3

The exterior angles of a quadrilateral are in the ratio 2 : 2 : 3 : 3. Find the magnitude of each exterior angle.

Sum of the exterior angles $= 360^\circ$

Ratio of the 4 angles $= 2 : 2 : 3 : 3$

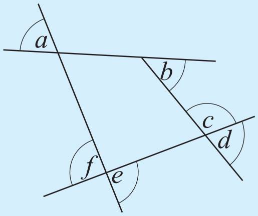
$$\therefore \text{ the smaller angles } = 360^\circ \times \frac{2}{10} = 72^\circ$$

$$\text{The larger angles } = 360^\circ \times \frac{3}{10} = 108^\circ$$

$\therefore$ the exterior angles are $72^\circ, 72^\circ,\ 108^\circ$ and 108°.

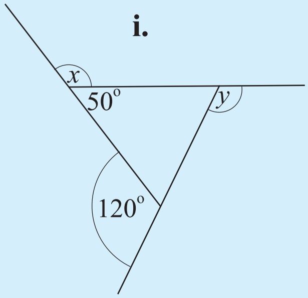
Exercise 25.2

1. From the angles denoted by a, b, c, d, e and f in the figure, select and write the ones which are exterior angles of the quadrilateral.

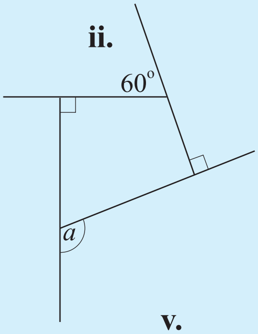


2. For each of the polygons given below, find the magnitude of the angle/angles denoted by the English letter/letters.

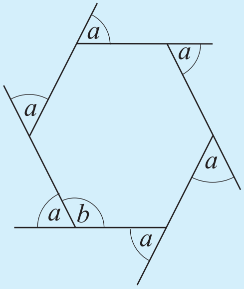
i.



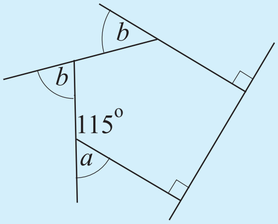
ii.



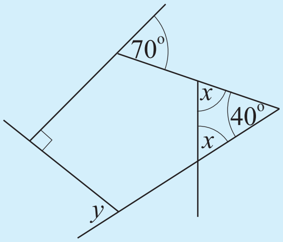
iii.



iv.



v.



3. The exterior angles of a quadrilateral are x°, $2x^\circ$, $3x^\circ$ and $4x^\circ$.
  i. Find the magnitude of each of the exterior angles.
  ii. Write the magnitude of each of the interior angles.

4. The exterior angles of a pentagon are in the ratio 1 : 1 : 2 : 3 : 3. Find the magnitude of each of the exterior angles.

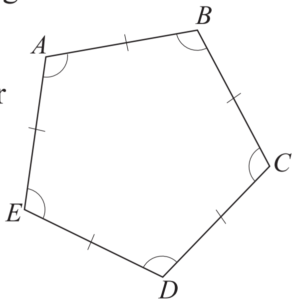
5. The exterior angles of a dodecagon are equal. Find the magnitude of any one of these exterior angles.

6. The magnitude of an exterior angle of a polygon whose exterior angles are equal is 18°. Find the number of sides the polygon has.

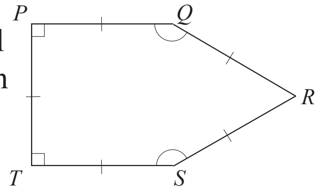
25.3 Regular Polygons

When all the sides of a polygon are equal in length and all the interior angles are equal in magnitude, the polygon is known as a regular polygon.

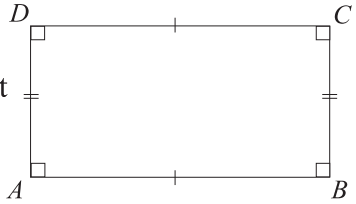
All the sides of the pentagon *ABCDE* in the figure are equal. All the interior angles are also equal. Therefore it is a regular pentagon.



All the sides of the pentagon *PQRST* are equal. However, all the interior angles are not equal. Therefore, the pentagon *PQRST* is not regular.



All the interior angles of the rectangle *ABCD* are equal. However, all the sides are not equal. Therefore, it is not a regular polygon.



Some regular polygons have special names. A regular triangle is called an **equilateral triangle**. A regular quadrilateral is called a **square**.

Example 1

Find the magnitude of an exterior angle of a regular hexagon and thereby find the magnitude of an interior angle.

$$\text{Sum of the six exterior angles} = 360^{\circ}$$

$$\therefore \text{ the magnitude of an exterior angle} = \frac{360^{\circ}}{6} = 60^{\circ}$$

$$\text{exterior angle} + \text{interior angle} = 180^{\circ}$$

$$\therefore 60^{\circ} + \text{interior angle} = 180^{\circ}$$

$$\therefore \text{ interior angle} = 180^{\circ} - 60^{\circ}$$

$$= \underline{\underline{120^{\circ}}}$$

Example 2

The magnitude of an interior angle of a regular polygon is 150°. Find,

- **i.** the magnitude of an exterior angle.
- **ii.** the number of sides.

i. $\text{exterior angle} + \text{interior angle} = 180^{\circ}$

$$\therefore \text{ exterior angle} + 150^{\circ} = 180^{\circ}$$

$$\therefore \text{ exterior angle} = 180^{\circ} - 150^{\circ} = \underline{\underline{30^{\circ}}}$$

ii. $\text{The number of sides} = \dfrac{360^{\circ}}{30^{\circ}} = \underline{\underline{12}}$

Exercise 25.3

1. Find the magnitude of an exterior angle of a regular pentagon and thereby find the magnitude of an interior angle.

2. Find the magnitude of an exterior angle of a regular polygon with 15 sides and thereby find the magnitude of an interior angle.

3. **i.** Find the number of sides of a regular polygon with an exterior angle of magnitude 120° and write the special name given to it.
   ii. Write with reasons the special name given to the regular polygon which has an exterior angle of magnitude 90°.
   iii. Write the name given to the regular polygon which has an exterior angle of magnitude 40°.

4. The magnitude of an interior angle of a regular polygon is four times the magnitude of an exterior angle. Find
   i. the magnitude of an exterior angle.
   ii. the magnitude of an interior angle.
   iii. the number of sides the polygon has.

5. What is the greatest value that an exterior angle of a regular polygon can be? What is the name given to the corresponding regular polygon?

Summary

- Sum of the exterior angles of a polygon = 360^{o}.
- The sum of the interior angles of a polygon of n sides $=$ $(2n - 4)$ right angles.

26

Algebraic Fractions

By studying this lesson you will be able to;

- identify algebraic fractions,
- add and subtract algebraic fractions with integral denominators (equal/ unequal denominators)
- add and subtract algebraic fractions with equal algebraic denominators.

We have already learnt to add and subtract numerical fractions and simplify, expand and factorize algebraic expressions.

Do the following review exercise to recall what has been learnt earlier.

Review Exercise

1. Simplify.

i. $\frac{2}{5}+\frac{1}{5}$ **ii.** $\frac{5}{7}-\frac{2}{7}$ **iii.** $\frac{1}{9}-\frac{7}{9}+\frac{4}{9}$ **iv.** $\frac{12}{13}-\frac{2}{13}-\frac{1}{13}$

2. Fill in each box with the appropriate number.

i. $\frac{1}{2}-\frac{1}{4}$

$=\frac{1\times\square}{2\times 2}-\frac{1}{4}$

$=\frac{\square\ -1}{4}$

$=\underline{\underline{\frac{\square}{4}}}$

ii. $\frac{3}{4}-\frac{2}{3}$

$=\frac{3\times\square}{4\times 3}-\frac{\square\times 4}{3\times 4}$

$=\frac{\square-\square}{12}$

$=\underline{\underline{\frac{\square}{12}}}$

iii. $\frac{4}{5}-\frac{3}{10}-\frac{1}{3}$

$=\frac{4\times\square}{5\times 6}-\frac{3\times\square}{10\times 3}-\frac{1\times 10}{3\times\square}$

$=\frac{\square-\square-10}{30}$

$=\frac{\square}{30}$

$=\frac{\square\ \div 5}{30\div 5}$

$=\underline{\underline{\frac{\square}{6}}}$

3. Simplify the algebraic expressions given below.

a. $2x + 3x$ **b.** $3y - y$
c. $5a + 4a + a$ **d.** $5x + 3y + x + 3y$
e. $3y + 2 - y - 2$ **f.** $4n - 1 + 5 - 2n$
g. $-3y + 2 - y - 3 + 2y$ **h.** $5xy - 6xy + 3x + y$

4. Expand and simplify.

a. $2(x + y) + 3x$ **b.** $3(2x - 4y) + 12y$
c. $-(4 - 3x) - 1$ **d.** $2(3x - 2) + 3(x + 2)$
e. $3(m + 1) - 2(2m - 1)$ **f.** $x(x - y) + 2xy$

5. For each of the statements given below, if it is true, mark a "✓" and if it is false, mark a "×" in the box to the right of the statement.

a. The value of $\frac{2}{3} + \frac{1}{4}$ is the same as the value of $\frac{2+1}{3+4}$. ☐

b. To obtain the sum or the difference of two fractions, their numerators should be equal; if they are not equal, then they should be made equal. ☐

c. The numerator of the sum of two unit fractions is the sum of the denominators of the original two fractions and the denominator is the product of the denominators of the original two fractions. ☐

d. When adding or subtracting two fractions with unequal denominators, the common denominator that should be used is the L. C. M. of the denominators of the original two fractions. ☐

e. By multiplying the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

f. By dividing the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

g. $-3x - 2x$ can be considered as $(-3x) + (-2x)$. ☐

h. To expand $-3(2x - 5)$, the terms $2x$ and -5 need to be multiplied by 3. ☐

i. When $-x - x$ is simplified we obtain $2x$. ☐

j. When $3x + 4y$ is simplified we obtain $7xy$. ☐

Introduction to algebraic fractions

If the numerator or denominator or both the numerator and the denominator of a fraction contain an algebraic term or expression, then that fraction is known as an algebraic fraction.

Example 1

Write 5 algebraic fractions that have an algebraic term only in the numerator.

$$\frac{x}{2}, \frac{3x}{5}, \frac{7y}{20}, \frac{6mn}{3}, \frac{2t^2}{5}$$

Example 2

Write 5 algebraic fractions that have an algebraic expression only in the numerator.

$$\frac{x+1}{5}, \frac{2x-1}{3}, \frac{x+y}{2}, \frac{m-n}{7}, \frac{3m-2n-1}{10}$$

Example 3

Write 5 algebraic fractions that have an algebraic term only in the denominator.

$$\frac{3}{x}, \frac{2}{3m}, \frac{5}{2y}, \frac{4}{3xy}, \frac{5}{m^2}$$

Example 4

Write 5 algebraic fractions that have an algebraic expression only in the denominator.

$$\frac{3}{2x+1}, \frac{2}{a+b}, \frac{5}{2m-n}, \frac{4}{3x-2y}, \frac{1}{3x+cy+2}$$

Example 5

Write 5 algebraic fractions which have an algebraic term in both the numerator and the denominator.

$$\frac{a}{c}, \frac{2a}{d}, \frac{2m}{3n}, \frac{4x}{5y}, \frac{2xy}{3pq}, \frac{2x^2}{5y^2}$$

Example 6

Write 5 algebraic fractions that have an algebraic expression in the numerator and an algebraic term in the denominator.

$$\frac{x+1}{2x}, \frac{2a+b}{c}, \frac{3a+d}{4a}, \frac{2x-1}{c}, \frac{4x^2y-a^2}{b}$$

Example 7

Write 5 algebraic fractions that have an algebraic term in the numerator and an algebraic expression in the denominator.

$$\frac{x}{2x+5}, \frac{a}{5b+d}, \frac{3c}{a+b}, \frac{4xy}{5x-3}, \frac{a^2}{a-b}$$

Example 8

Write 5 algebraic fractions that have algebraic expressions in both the numerator and the denominator.

$$\frac{x+1}{2x-1}, \frac{x+y}{3x+2y}, \frac{3x-4}{x+1}, \frac{4m-3n}{5m+2n}, \frac{4x-y}{2x+3y-4}$$

26.1 Adding and subtracting algebraic fractions with equal integral denominators

We can add and subtract algebraic fractions in the same way that we added and subtracted fractions with whole numbers in the numerator and denominator.

Example 1

Express $\frac{5x}{9} + \frac{2x}{9}$ as a single fraction.

$$\frac{5x}{9} + \frac{2x}{9} = \frac{5x+2x}{9} \quad \text{(since the denominators of both fractions are equal)}$$

$$= \underline{\underline{\frac{7x}{9}}}$$

Example 2

Simplify $\frac{5y}{7} - \frac{3y}{7}$.

$\frac{5y}{7} - \frac{3y}{7} = \frac{5y - 3y}{7}$ (since the denominators of both fractions are equal)

$= \underline{\underline{\frac{2y}{7}}}$

Example 3

Simplify $\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15}$.

$\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15} = \frac{11x - 2x}{15}$ (since the denominators of both fractions are equal)

$= \frac{9x}{15} = \underline{\underline{\frac{3x}{5}}}$ (by dividing by 3, the highest common factor of 9 and 15)

Example 4

Simplify $\frac{x + 1}{5} + \frac{x + 2}{5}$.

$\frac{x + 1}{5} + \frac{x + 2}{5} = \frac{x + 1 + x + 2}{5}$ (since the denominators of both fractions are equal)

$= \frac{x + x + 1 + 2}{5}$

$= \underline{\underline{\frac{2x + 3}{5}}}$

Example 5

Simplify $\frac{2b + 3}{7} - \frac{b + 2}{7}$.

$\frac{2b + 3}{7} - \frac{b + 2}{7} = \frac{2b + 3 - (b + 2)}{7}$ (the algebraic expression to be subtracted must be written within brackets)

$= \frac{2b + 3 - b - 2}{7}$

$= \frac{2b - b + 3 - 2}{7}$

$= \underline{\underline{\frac{b + 1}{7}}}$

Example 6

Simplify $\frac{7c+1}{8} - \frac{2c+1}{8} - \frac{c-2}{8}$.

$$\begin{aligned}\frac{7c+1}{8} - \frac{2c+1}{8} - \frac{c-2}{8} &= \frac{7c+1-(2c+1)-(c-2)}{8}\\ &= \frac{7c+1-2c-1-c+2}{8}\\ &= \frac{4c+2}{8}\\ &= \frac{2\,(2c+1)}{8}\\ &= \underline{\underline{\frac{2c+1}{4}}}\end{aligned}$$

Exercise 26.1

1. Simplify and write the answer in the simplest form.

a. $\frac{a}{5} + \frac{a}{5}$
b. $\frac{3d}{15} + \frac{2d}{15}$
c. $\frac{2t}{3} - \frac{t}{3}$
d. $\frac{7k}{8} - \frac{3k}{8}$
e. $\frac{3k}{7} + \frac{2k}{7} + \frac{k}{7}$
f. $\frac{5h}{9} - \frac{2h}{9} - \frac{h}{9}$
g. $\frac{7v}{10} - \frac{3v}{10} + \frac{v}{10}$
h. $\frac{x}{8} - \frac{3x}{8}$
i. $\frac{p}{9} - \frac{4q}{9} - \frac{5p}{9}$

2. Simplify and write the answer in the simplest form.

a. $\frac{3y+1}{5} + \frac{2y+2}{5}$
b. $\frac{4m-1}{7} + \frac{3m-2}{7}$
c. $\frac{5n+3}{8} + \frac{2n-1}{8}$
d. $\frac{5c-2}{10} + \frac{3c+4}{10}$
e. $\frac{6d+1}{10} - \frac{2d-3}{10}$
f. $\frac{3x+1}{6} - \frac{2x-3}{6} + \frac{x+4}{6}$

26.2 Adding and subtracting algebraic fractions with unequal integral denominators

Now let us consider how to simplify expressions of algebraic fractions with unequal integral denominators such as $\frac{x}{6} + \frac{3x}{4}$. These types of fractions can be simplified in the same way that numerical fractions are simplified. A common multiple of the denominators of the fractions can be taken as the common denominator. However simplification is made easier by taking the least common multiple of the denominators.

For example, the denominators of the above two fractions are 6 and 4. Their least common multiple is 12. Therefore, initially, the above fractions need to be converted into fractions with denominator 12. To convert $\frac{x}{6}$ into a fraction with denominator 12, the denominator and numerator of $\frac{x}{6}$ need to be multiplied by 2. (Observe that 2 is obtained from $\frac{12}{6}$). Similarly, to convert $\frac{3x}{4}$ into a fraction with denominator 12, we need to multiply the numerator and denominator of $\frac{3x}{4}$ by 3. (Observe that 3 is obtained from $\frac{12}{4}$). Accordingly, we may write the following to simplify the given expression.

$$\frac{x}{6} + \frac{3x}{4} = \frac{2}{2} \times \frac{x}{6} + \frac{3}{3} \times \frac{3x}{4}$$

When we simplify the numerator and the denominator of these fractions we obtain the following.

$$\frac{2x}{12} + \frac{9x}{12}$$

Now since both fractions have a common denominator, we can write the above as follows.

$$\frac{2x + 9x}{12}$$

By simplifying this we get $\frac{11x}{12}$.

Accordingly, $\frac{x}{6} + \frac{3x}{4} = \frac{11x}{12}$.

Example 1

Simplify $\frac{2y}{5} + \frac{y}{4}$.

$$\frac{2y}{5} + \frac{y}{4} = \frac{4 \times 2y}{4 \times 5} + \frac{5 \times y}{5 \times 4}$$

(Since the L. C. M. of 5 and 4 is 20, equivalent fractions with 20 as the denominator are obtained.)

$$= \frac{8y}{20} + \frac{5y}{20}$$

$$= \frac{8y + 5y}{20} = \underline{\underline{\frac{13y}{20}}}$$

Example 2

Simplify $\frac{2t}{3} - \frac{t}{2}$.

$$\frac{2t}{3} - \frac{t}{2} = \frac{2 \times 2t}{2 \times 3} - \frac{3 \times t}{3 \times 2}$$

(since the L.C.M. of 3 and 2 is 6, equivalent fractions with 6 as the denominator are obtained)

$$= \frac{4t}{6} - \frac{3t}{6}$$

$$= \frac{4t - 3t}{6}$$

$$= \underline{\underline{\frac{t}{6}}}$$

Example 3

Simplify $\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4}$.

$$\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4} = \frac{10 \times 3v}{10 \times 2} - \frac{4 \times 4v}{4 \times 5} + \frac{5 \times 3v}{5 \times 4}$$

(since the L.C.M. of 2, 4 and 5 is 20, equivalent fractions with 20 as the denominator are obtained)

$$= \frac{30v}{20} - \frac{16v}{20} + \frac{15v}{20}$$

$$= \underline{\underline{\frac{29v}{20}}}$$

You may have observed in the above examples that when the denominators are not equal, it is easy to simplify by taking the L.C.M. of the unequal denominators as the common denominator.

Now let us consider instances where we have to multiply an algebraic expression by a number. Here it is important to remember to write the algebraic expression within brackets.

Example 4

Simplify $\frac{x + 1}{2} + \frac{2x + 1}{3}$.

$$\frac{x + 1}{2} + \frac{2x + 1}{3} = \frac{3\,(x + 1)}{3 \times 2} + \frac{2\,(2x + 1)}{2 \times 3}$$

(writing the algebraic expression within brackets; L.C.M. of 2 and 3 is 6)

$$= \frac{3x + 3}{6} + \frac{4x + 2}{6} \qquad \text{(expanding)}$$

$$= \frac{7x + 5}{6}$$

Example 5

$$\frac{5y - 1}{6} - \frac{3y - 2}{4}$$

$$\frac{5y - 1}{6} - \frac{3y - 2}{4} = \frac{2\,(5y - 1)}{2 \times 6} - \frac{3\,(3y - 2)}{3 \times 4} \quad \text{(L.C.M. of 4 and 6 is 12)}$$

$$= \frac{2\,(5y - 1)}{12} - \frac{3\,(3y - 2)}{12}$$

$$= \frac{2\,(5y - 1) - 3\,(3y - 2)}{12}$$

$$= \frac{10y - 2 - 9y + 6}{12} \quad \text{(expanding by multiplying by 2 and by − 3)}$$

$$= \frac{y + 4}{12}$$

Example 6

$$\frac{3m + 2n}{5} - \frac{2m - n}{10} - \frac{3m - 2n}{15}$$

$$= \frac{6\,(3m + 2n)}{6 \times 5} - \frac{3\,(2m - n)}{3 \times 10} - \frac{2\,(3m - 2n)}{2 \times 15} \quad \text{(L.C.M. of 5, 10 and 15 is 30)}$$

$$= \frac{6\,(3m + 2n)}{30} - \frac{3\,(2m - n)}{30} - \frac{2\,(3m - 2n)}{30}$$

$$= \frac{18m + 12n - 6m + 3n - 6m + 4n}{30}$$

$$= \frac{6m + 19n}{30}$$

Exercise 26.2

1. Simplify and give the answer in the simplest form.

a. $\frac{a}{3} + \frac{a}{6}$ **b.** $\frac{b}{4} + \frac{b}{12}$ **c.** $\frac{5x}{3} - \frac{x}{6}$

d. $\frac{3y}{4} - \frac{5y}{16}$ **e.** $\frac{a}{2} + \frac{a}{3}$ **f.** $\frac{c}{3} - \frac{c}{4}$

g. $\frac{3n}{7} + \frac{n}{5}$ **h.** $\frac{3d}{10} + \frac{2d}{15}$ **i.** $\frac{5m}{6} - \frac{3m}{10}$

2. Simplify and give the answer in the simplest form.

a. $\frac{a}{2} + \frac{a}{3} + \frac{a}{4}$ **b.** $\frac{c}{5} + \frac{3c}{10} + \frac{2c}{15}$

c. $\frac{3x}{5} + \frac{x}{6} - \frac{2x}{15}$ **d.** $\frac{3n}{4} - \frac{3n}{8} - \frac{n}{2}$

3. Simplify and write in the simplest form.

a. $\frac{2a}{5} + \frac{3a-2}{6}$ **b.** $\frac{2b-1}{8} + \frac{3b}{12}$

c. $\frac{3c+2}{6} + \frac{2c-1}{9}$ **d.** $\frac{5t-3}{10} - \frac{3t}{15}$

e. $\frac{2m-n}{12} - \frac{3m+n}{9}$ **f.** $\frac{3y+1}{10} + \frac{2y-1}{5} + \frac{4-y}{20}$

g. $\frac{3x-y}{4} + \frac{2x+y}{6} - \frac{5x-2y}{3}$ **h.** $\frac{3y+2}{3} - \frac{y-1}{4} - \frac{2y-3}{8}$

26.3 Adding and subtracting algebraic fractions with the same algebraic denominator

As an example of this type of algebraic fraction we have $\frac{2}{5x} + \frac{1}{5x}$. Although the denominators of these fractions are algebraic terms, since they are equal, we can simplify this in the same way that we simplify numerical fractions.

Accordingly, we can simplify the above as,

$$\frac{2}{5x} + \frac{1}{5x} = \frac{2+1}{5x}$$

$$= \frac{3}{5x}$$

Example 1

Simplify $\frac{4}{7m} + \frac{2}{7m}$.

$$\frac{4}{7m} + \frac{2}{7m} = \frac{4+2}{7m}$$

$$= \underline{\underline{\frac{6}{7m}}}$$

Example 2

Simplify $\frac{5}{6n} - \frac{1}{6n}$.

$$\frac{5}{6n} - \frac{1}{6n} = \frac{5-1}{6n}$$

$$= \frac{4}{6n}$$

$$= \underline{\underline{\frac{2}{3n}}}$$

(simplifying by dividing by the common factor 2)

Example 3

Simplify $\frac{3a}{4b} + \frac{1}{4b} - \frac{a}{4b}$.

$$\frac{3a}{4b} + \frac{1}{4b} - \frac{a}{4b} = \frac{3a+1-a}{4b}$$ (common denominator is $4b$)

$$= \underline{\underline{\frac{2a+1}{4b}}}$$

Example 4

Simplify $\frac{3}{x+1} + \frac{2}{x+1}$.

Although the denominators are algebraic expressions, since they are equal, this can be simplified in the same manner as above.

$$\frac{3}{x+1} + \frac{2}{x+1} = \frac{3+2}{x+1}$$

$$= \underline{\underline{\frac{5}{x+1}}}$$

Example 5

Simplify $\frac{7}{x-3} - \frac{4}{x-3}$.

$$\frac{7}{x-3} - \frac{4}{x-3} = \frac{7-4}{x-3}$$ (common denominator is $x - 3$)

$$= \underline{\underline{\frac{3}{x-3}}}$$

Exercise 26.3

1. Simplify and give the answer in the simplest form.

a. $\dfrac{5}{a} + \dfrac{2}{a}$ **b.** $\dfrac{8}{x} + \dfrac{2}{x}$ **c.** $\dfrac{3}{y} - \dfrac{1}{y}$

d. $\dfrac{4}{3y} - \dfrac{2}{3y}$ **e.** $\dfrac{3}{5t} + \dfrac{2}{5t}$ **f.** $\dfrac{h}{2k} + \dfrac{5h}{2k}$

g. $\dfrac{7}{2n} + \dfrac{3}{2n} - \dfrac{1}{2n}$ **h.** $\dfrac{8}{3v} - \dfrac{4}{3v} - \dfrac{1}{3v}$ **i.** $\dfrac{5}{m} + \dfrac{2}{m} + \dfrac{1}{m}$

j. $\dfrac{8}{7xy} - \dfrac{8}{7xy} + \dfrac{8}{7xy}$

2. Simplify and give the answer in the simplest form.

a. $\dfrac{5}{m+3} + \dfrac{2}{m+3}$ **b.** $\dfrac{8}{n+5} + \dfrac{3}{n+5}$ **c.** $\dfrac{4}{a+b} + \dfrac{6}{a+b}$

d. $\dfrac{4x}{x+2y} + \dfrac{x+y}{x+2y}$ **e.** $\dfrac{9h}{x+y} - \dfrac{7h-2}{x+y}$ **f.** $\dfrac{3x+y}{x-3y} - \dfrac{2x+4y}{x-3y}$

26.4 Simplifying algebraic fractions with algebraic expressions in the numerator and the denominator

Example 1

Simplify $\dfrac{5x}{2x+1} + \dfrac{3x}{2x+1}$.

$\dfrac{5x}{2x+1} + \dfrac{3x}{2x+1} = \dfrac{5x+3x}{2x+1}$ (the common denominator is $2x + 1$)

$= \dfrac{8x}{2x+1}$

Example 2

Simplify $\dfrac{7y}{3y-1} - \dfrac{2y}{3y-1}$.

$\dfrac{7y}{3y-1} - \dfrac{2y}{3y-1} = \dfrac{7y-2y}{3y-1}$ (the common denominator is $3y - 1$)

$= \dfrac{5y}{3y-1}$

Example 3

Simplify $\frac{2x-1}{5x+1} + \frac{3x+2}{5x+1}$.

$$\frac{2x-1}{5x+1} + \frac{3x+2}{5x+1} = \frac{2x-1+3x+2}{5x+1} \quad \text{(the common denominator is } 5x+1\text{)}$$

$$= \frac{5x+1}{5x+1}$$

$$= \underline{\underline{1}}$$

Example 4

Simplify $\frac{9m-1}{5m-1} + \frac{3m}{5m-1} - \frac{2m+1}{5m-1}$.

$$\frac{9m-1}{5m-1} + \frac{3m}{5m-1} - \frac{2m+1}{5m-1} = \frac{9m-1+3m-(2m+1)}{5m-1}$$

(algebraic expressions to be subtracted need to be written within brackets)

$$= \frac{9m-1+3m-2m-1}{5m-1}$$

(multiplying by the – sign and expanding)

$$= \frac{10m-2}{5m-1}$$

$$= \frac{2\cancel{(5m-1)}}{\cancel{(5m-1)}}$$

(separating out the common factor in the numerator and simplifying)

$$= \underline{\underline{2}}$$

Exercise 26.4

1. Simplify and write the answer in the simplest form.

a. $\frac{k}{3k-1} + \frac{2}{3k-1}$

b. $\frac{2h}{5h-2} - \frac{h}{5h-2}$

c. $\frac{3t}{3t-1} - \frac{1}{3t-1}$

d. $\frac{2k+1}{5k+1} - \frac{k-2}{5k+1}$

e. $\dfrac{2y}{3y+2} - \dfrac{y}{3y+2} + \dfrac{1}{3y+2}$

f. $\dfrac{2a+1}{5a-2} - \dfrac{3a}{5a-2} - \dfrac{3}{5a-2}$

g. $\dfrac{8m+10}{2m+3} - \dfrac{4m+1}{2m+3} + \dfrac{2m}{2m+3}$

h. $\dfrac{m}{m+n} - \dfrac{m-n}{m+n} - \dfrac{m-n}{m+n}$

27

Scale Diagrams

By studying this lesson you will be able to;

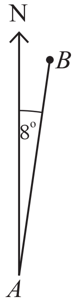
- identify bearings,
- draw a scale diagram of locations in a horizontal plane when bearings and distances are given, and find unknown quantities using the scale diagram.

27.1 Bearing

Bearing is a measurement that is used to indicate a direction in a horizontal plane.

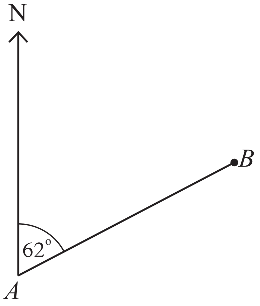
The bearing of the point *B* from the point *A* is the angle that the direction *AB* makes with the direction of North when measured from *A* in a clockwise direction. The following figures illustrate the bearing of *B* from *A* for different locations of *A* and *B*. Observe that the bearing is given in three digits.

i.



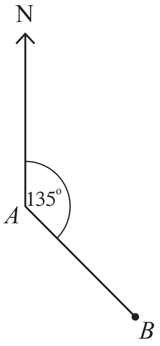
The bearing of *B* from *A* = 008°

ii.



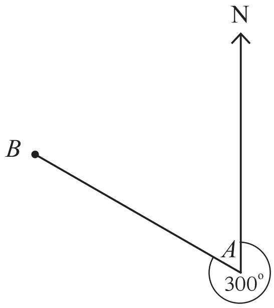
The bearing of *B* from *A* = 062°

iv.



The bearing of *B* from *A* = 135°

v.



The bearing of *B* from *A* = 300°

Since a bearing is always less than 360°, the maximum number of digits it can have is three. Therefore the norm is to always write bearings with three digits. If the angle is one of $1^\circ, 2^\circ, \ldots, 9^\circ$, then the bearing is written as $001^\circ, 002^\circ, \ldots, 009^\circ$ and if the angle is one of $10^\circ, 11^\circ, \ldots, 99^\circ$, then it is written as $010^\circ, 011^\circ, \ldots, 099^\circ$.

Accordingly bearing is,

i. measured starting from the North,
ii. measured in a clockwise direction,
iii. written with three digits.

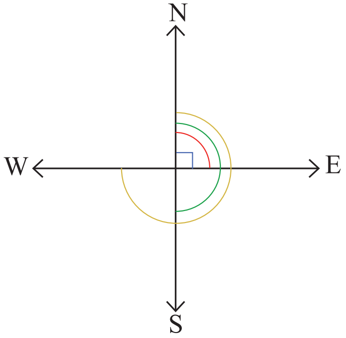
Since the North can be easily identified using a compass, bearings are used widely in sea and air travel.

Let us broaden our knowledge on bearings by studying the examples given below.

Example 1

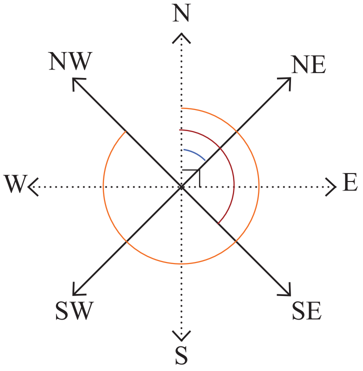
i. Indicate the four main directions (cardinal directions) in terms of bearings.
ii. Indicate the four sub-directions (intermediate directions) in terms of bearings.

i.



Direction	Bearing
North	000°
East	090°
South	180°
West	270°

ii.

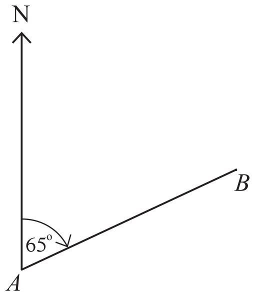


Direction	Bearing
Northeast	045°
Southeast	135°
Southwest	225°
Northwest	315°

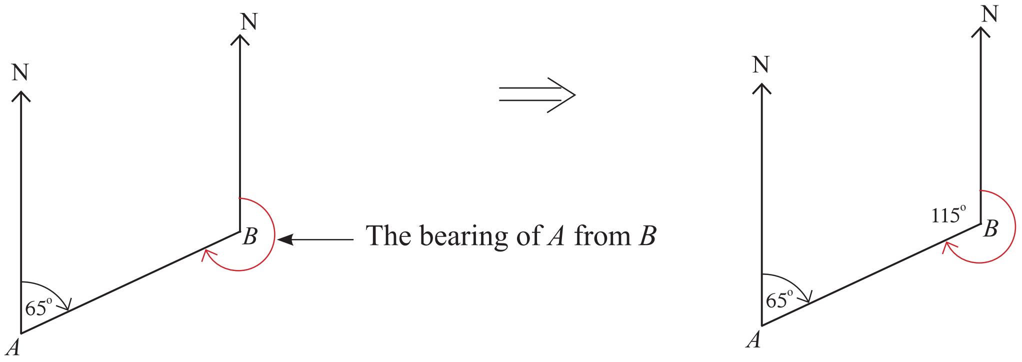
Example 2

The bearing of *B* from *A* is 065°. Illustrate this information in a rough sketch and find the bearing of *A* from *B*.

Since the bearing of *B* from *A* is 065°, the angle drawn from the direction of North at *A* to the directionof *AB* in the clockwise direction is 65°.



Now, to find the bearing of *A* from *B*, a line needs to be drawn in the direction of North from *B*, and the angle that is formed when this line is rotated in a clockwise direction about *B* from the direction of North to the direction of *BA* needs to be found.



The lines drawn at *A* and *B* in the direction of North are parallel. The pair of allied angles formed by the transversal *AB* intersecting these lines are supplementary. Using this fact, the value 115° has been found. The required bearing is indicated in the figure given above.

As the sum of the angles around a point is 360°,

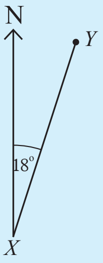
the bearing of *A* from *B* $= 360^\circ - 115^\circ$

$= \underline{\underline{245^\circ}}$

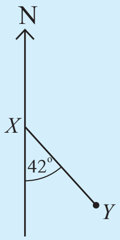
Exercise 27.1

1. In each of the following situations, find the bearing of Y from X.

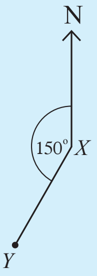
i.



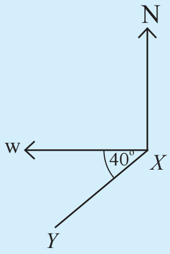
ii.



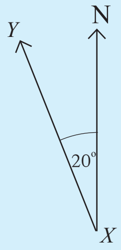
iii.



iv.



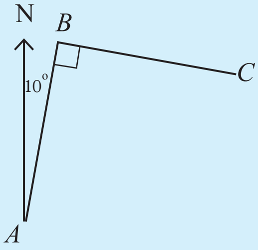
v.



2. Using a protractor to measure the angles, illustrate each of the following bearings by drawing a figure.

i. The bearing of F from E is 005°.
ii. The bearing of Q from P is 075°.
iii. The bearing of N from M is 105°.
iv. The bearing of H from J is 270°.
v. The bearing of D from C is 310°.

3.



Based on the information given in the figure,

i. determine the bearing of B from A,
ii. determine the bearing of A from B,
iii. determine the bearing of B from C.

4. ABC is an equilateral triangle. B is situated to the north of A.

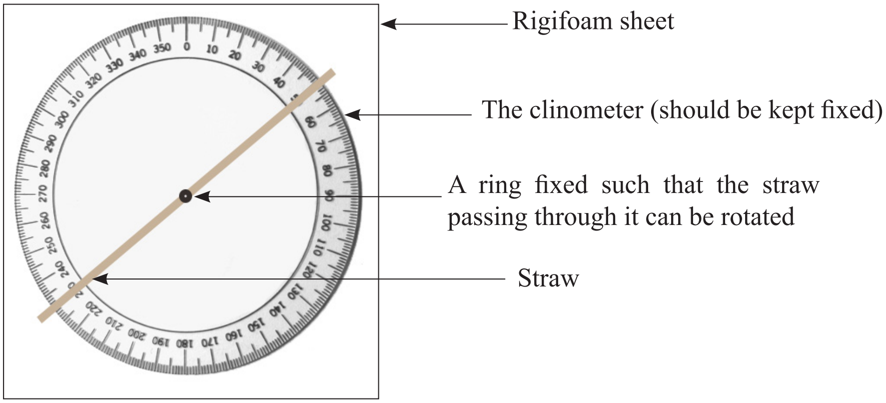
i. Illustrate this information by a rough sketch.
ii. By considering the sketch determine the following.

a. Bearing of B from A
b. Bearing of C from A
c. Bearing of C from B
d. Bearing of B from C
e. Bearing of A from C
f. Bearing of A from B

27.2 Clinometer

Any location in a horizontal plane can be described in terms of bearings and distances. A clinometer can be used to determine bearings.

Clinometer



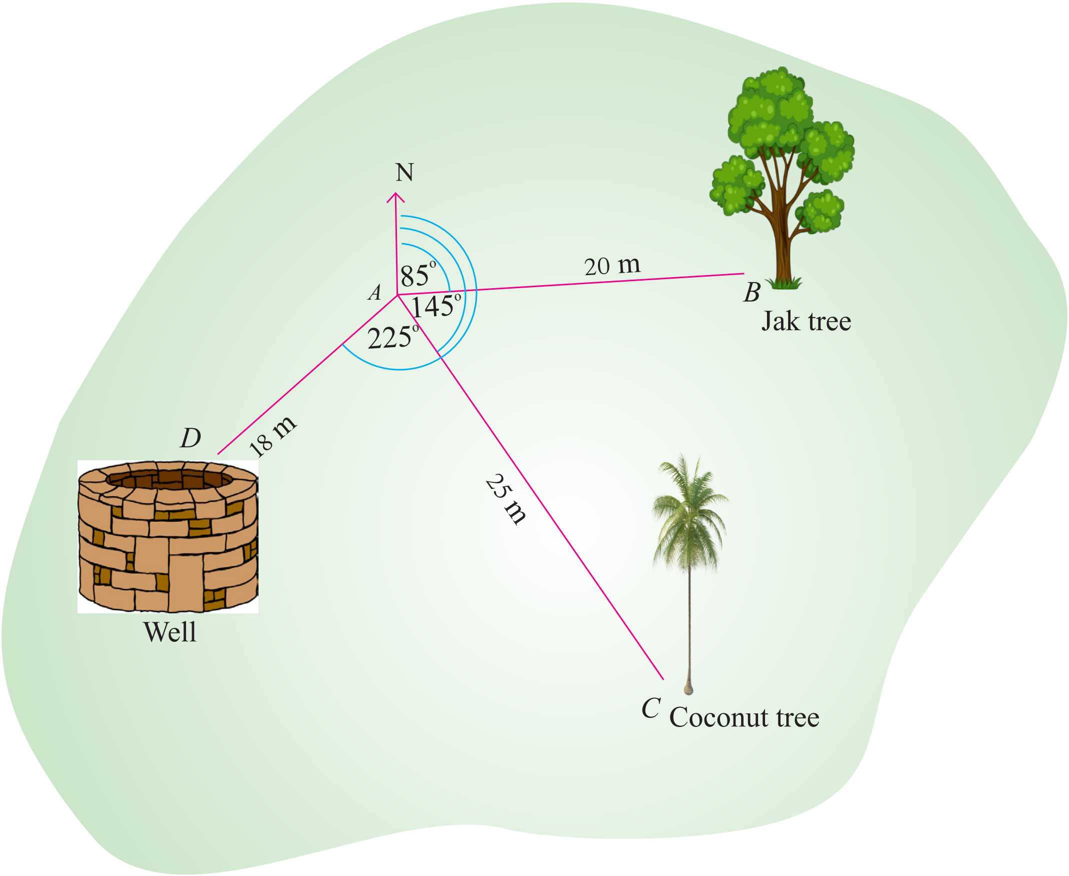
- Place a compass on the horizontal tabletop of a table kept at *A*. Suppose for example that we want to describe the location of *B* with respect to location *A* and mark the direction of North on the tabletop.

- Place the clinometer on the tabletop such that the "0" on the clinometer is towards the North.

- Rotate the straw until the location *B* is observed through the straw and measure the clockwise angle of rotation from the direction of North. By writing it using three digits, the bearing of *B* is obtained.

- By measuring the distance from *A* to *B* using a measuring tape, the position of *B* can be described in terms of the distance and bearing from *A*.

In the following figure the bearings of *B*, *C* and *D* with respect to *A* are given.

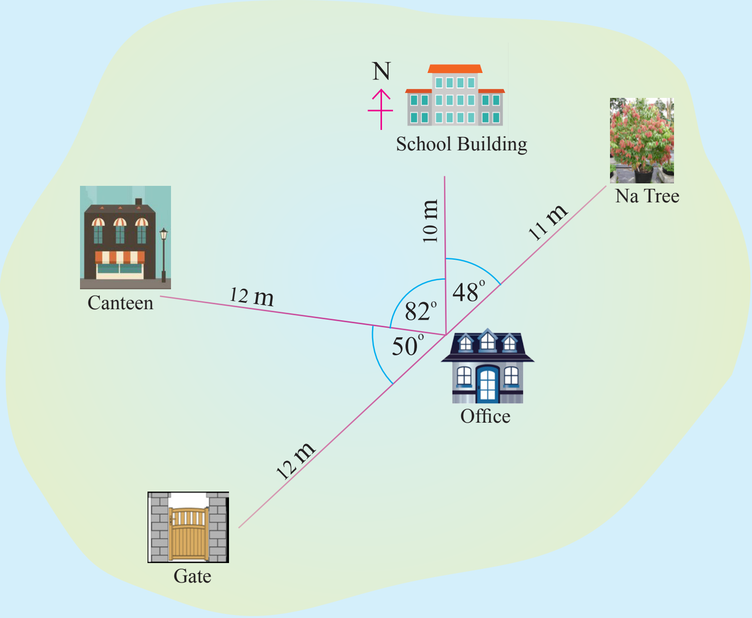


Object which was observed	Bearing	Distance
Jak tree (*B*)	085°	20 m
Coconut tree (*C*)	145°	25 m
Well (*D*)	225°	18 m

Do the following exercise to broaden your knowledge on this topic.

Exercise 27.2

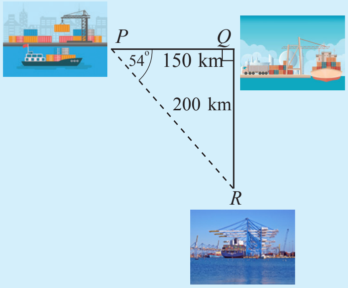
1. A rough school plan is shown below.



Using it describe the following.

i. The location of the Na tree with respect to the School Office
ii. The location of the gate with respect to the School Office
iii. The location of the Canteen with respect to the School Office

2.



P, *Q* and *R* denote three harbours located in the same ocean. *Q* is to the East of *P*. Describe the route in terms of the bearing and distance that a ship needs to take to journey,

i. from harbour *P* to harbour *R* through *Q*.
ii. directly from harbour *P* to harbour *R*.

3. A pilot of a certain air plane which is scheduled to fly from Colombo to a certain airport has been instructed to fly 100 km on a bearing of 020° and then another 100 km on a bearing of 080°.

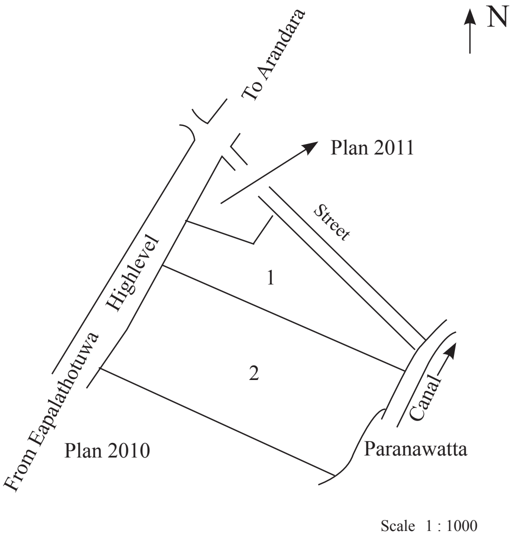
i. Represent this information in a rough sketch

ii. Write a description of the route that the pilot needs to take if he is to fly back to Colombo from that airport along the same path.

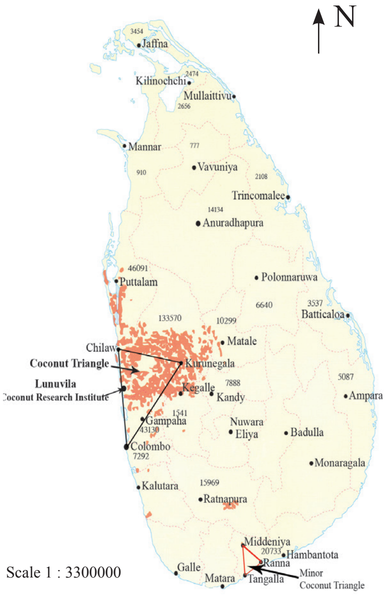
27.3 Scale diagrams in a horizontal plane

Given below are two examples of scale diagrams in a horizontal plane.

Survey Plane No: 103



Coconut Cultivation



In every scale diagram, the scale to which the diagram is drawn is given, and the direction of North is marked. It is very important to understand what is meant by the scale (ratio) given in the scale diagram. For example, a scale of 1 : 500 000 means that a distance of 500 000 cm is represented by 1 cm in the scale diagram. In other words, the distance between two points on the scale diagram is $\frac{1}{500\,000}$th of the actual distance between the two points. Moreover, since 500 000 cm is equal to 5 km, the actual distance represented by 1 cm in the scale diagram is 5 km.

Now let us learn how to draw scale diagrams by considering some examples.

Example 1

The vertices of a triangular floor area are A, B and C. The positions of the vertices with respect to a point P located in this area is given below.

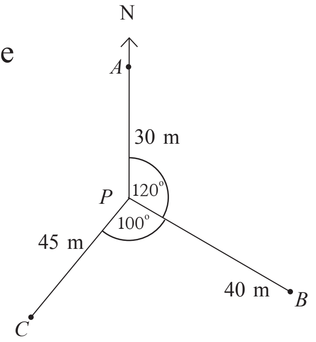
With respect to P,

- *A* is located 30 m away on a bearing of 000°
- *B* is located 40 m away on a bearing of 120°
- *C* is located 45 m away on a bearing of 220°

Draw a scale diagram of the floor area using this information and find its perimeter.

Step 1: Mark the direction of North at the top right hand corner of the sheet of paper.

Step 2: Draw a rough sketch as shown, based on the information that is given.



Step 3: To represent the distances 30 m, 40 m and 45 m, select the scale of 1 cm representing 10 m, that is, the scale of 1:1000.(Here, the scale should be selected according to the size of the sheet of paper. Moreover, by selecting a value such as 1000, anyone who is examining the scale diagram can easily get an idea of the actual distances represented in it.)

Step 4: For each distance that is to be represented in the scale diagram, calculate the corresponding length using the selected scale.

$$PA = 3000 \times \frac{1}{1000} \text{ cm} = 3 \text{ cm}$$

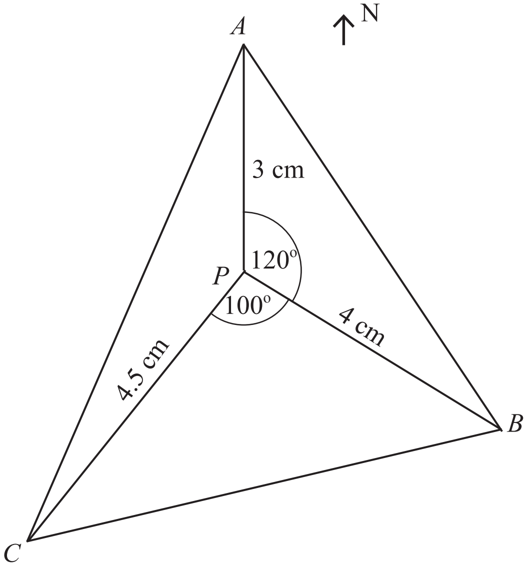
$$PB = 4000 \times \frac{1}{1000} \text{ cm} = 4 \text{ cm},$$

$$PC = 4500 \times \frac{1}{1000} \text{ cm} = 4.5 \text{ cm}$$

Step 5: Using a straight edge with a cm scale and a protractor, draw the scale diagram with a pencil as shown below.

- First draw the line segment *AP* of length 3cm upwards.
- Draw the line segment *PB* of length 4 cm which makes an angle of 120° clockwise with *PA*.
- Draw the line segment *PC* of length 4.5 cm which makes an angle of 100° clockwisewith *PB*.

- Draw the line segments *AB, BC* and *AC*.



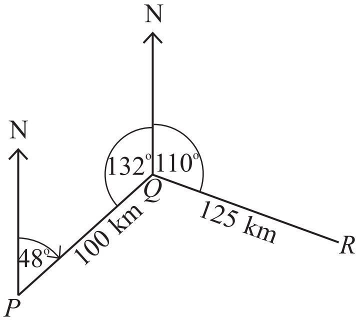
Step 6: Measure the lengths of *AB*, *BC* and *AC*. You will see that *AB* = 6 cm, *AC* = 7.1 cm and *BC* = 6.5 cm. Therefore the perimeter of the scale diagram is 6 + 7.1 + 6.5 = 19.6 cm.

Step 7: Using the scale 1 cm ⟶ 10 m, calculate the actual length.
Perimeter of the floor = 10 × 19.6 = 196 m

Example 2

A ship journeying from harbour *P* approaches harbour *Q* after travelling 100 km on a bearing of 048°. It then travels 125 km on a bearing of 110° and approaches harbour *R*. Draw a scale diagram and describe the position of *R* with respect to *P*.

Step 1: Based on the information given, draw a rough sketch as shown below.



Step 2: Mark a point *P* on a sheet of paper and mark the direction of North upwards.

- Since the bearing of Q from P is 048°, the angle that PQ makes with the direction of North at P is 48° in the clockwise direction.
- Since the bearing of R from Q is 110°, the angle that QR makes with the direction of North at Q is 110° in the clockwise direction.

Since the direction of North at P and the direction of North at Q are parallel, the angle formed between the direction of North at Q and PQ is 132° (allied angles)

$$\begin{aligned}\text{Therefore, } P\hat{Q}R &= 360^{\circ} - (132^{\circ} + 110^{\circ})\\ &= 360^{\circ} - 242^{\circ}\\ &= 118^{\circ}\end{aligned}$$

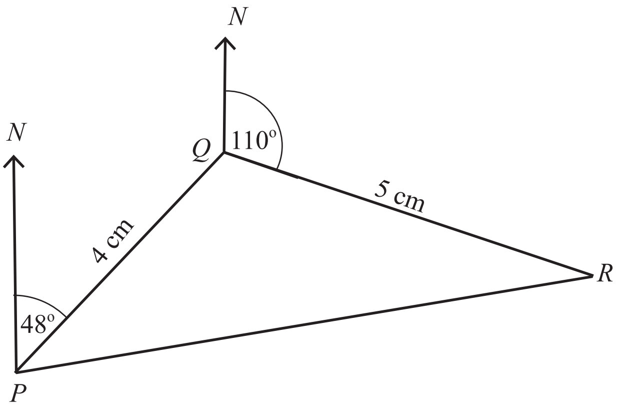
Step 3: Select the scale of 1 cm representing 25 km, that is, 1: 2 500 000 to represent the distances 100 km and 125 km in the scale diagram. (If there is space on the sheet of paper, the scale of 1:1 250 000 can also be used).

Step 4: According to the selected scale, calculate the lengths of by which PQ and QR are to be represented in the scale diagram.

$$PQ = \frac{100}{25}\text{ cm} = 4\text{ cm},\quad QR = \frac{125}{25}\text{ cm} = 5\text{ cm}$$

(When drawing scale diagrams, the magnitudes of the angles do not change.)

Step 5: Draw the scale diagram using a straight edge, a protractor and a pencil, based on the above measurements.



Step 6: When PR is measured, we obtain $PR = 7.7$ cm. When $N\hat{P}R$ is measured, we obtain $N\hat{P}R = 82^{\circ}$.

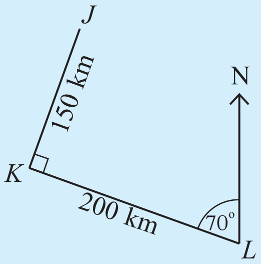
Step 7: Using the scale, calculate the actual length of PR.

$$\begin{aligned}\text{Actual length of } PR &= 7.7 \times 25\text{ km}\\ &= 192.5\text{ km}\end{aligned}$$

Step 8: The position of *R* can be described as follows.
R is situated 192.5 km from *P* on a bearing of 082°.

Exercise 27.3

1. A rough sketch of the route of a ship travelling from harbour *L* to harbour *K* and then from harbour *K* to harbour *J* is given.



i. Find the following based on this rough sketch.

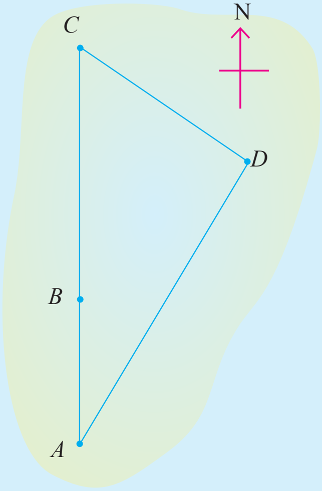
- **a.** Bearing of *K* from *L*
- **b.** Bearing of *J* from *K*
- **c.** The lengths of *LK* and *KJ* in a scale diagram drawn to the scale of 1 cm representing 50 km.

ii. Using the above scale, draw a scale diagram of the route of the ship.

iii. Using the scale diagram,
- **a.** find the distance from harbour *L* to harbour *J*
- **b.** find the bearing of harbour *L* from harbour *J*.

vi. Using the Pythagorean relation, calculate the distance from harbour *L* to harbour.
J and check whether the answer you obtained in (iii) (a) above is correct.

2.



A portion of a map drawn to the scale of 1 : 50 000 is shown here. While the cities *A*, *B* and *C* are situated on the same straight line, *C* lies directly north of *A*.

i. Measure and write the lengths of the line segments *AB*, *BC*, *CD* and *AD* and the magnitudes of the angles $A\hat{C}D$, $A\hat{D}C$ and $C\hat{A}D$.

ii. Calculate the actual distances of *AB, BC, CD* and *AD*.

iii. Describe the locations of *B, C* and *D* with respect to *A* in terms of bearings and distances from *A*.

3. The School Office is located at a distance of 10 m and on a bearing of 025° from the school flag post. The Main Hall is located at a distance of 12 m and on a bearing of 310° from the school flag post.

i. Draw a rough sketch based on the above information.

ii. Draw a scale diagram based on the sketch, using the scale of 1 cm representing 2 m.

iii. Using the scale diagram, find the shortest distance between the Office and the Main Hall.

iv. Describe the location of the Main Hall with respect to the Office.

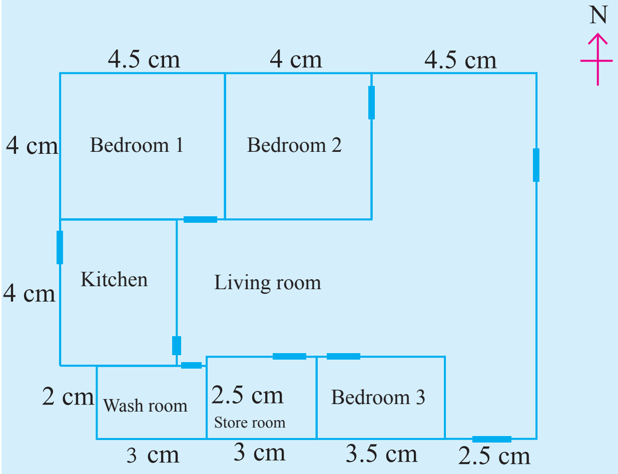
4. A pilot flies a plane 80 km on a bearing of 150^o and then 150 km on a bearing of 200^o and arrives at airport *B* from airport *A*.

i. Draw a rough sketch based on the above information.

ii. Draw a scale diagram using a suitable scale and find,

- **a.** the bearing of *B* from *A*
- **b.** the distance from *A* to *B*
- **c.** the bearing of *A* from *B*.

5. The floor plan of a house to be constructed, which is drawn to scale is shown below. Answer the questions given below using the scale diagram.



i. If the actual length of bedroom 2 is 4 m, express the scale to which this plan is drawn, as a ratio.

ii. Find the actual breadth of the house.

iii. Find the actual area of the washroom in square meters.

6. A person standing on a straight road that runs from East to West across a carnival ground, observes a flag post on a bearing of 115^o. When he travels 220 m to the East along the road, he see the flag post on a bearing of 210^o.

i. Describe the final location of the person with respect to the flag post.

ii. By drawing a scale diagram, find the shortest distance from the flag post to the road.

7. A portion of a road map of Sri Lanka drawn to the scale of 1 : 1 000 000 is shown here. The main "A" road is highlighted in red.

   - **i.** Find the actual length in kilometres that is represented by 1 cm in the map.
   - **ii.** With the aid of a string, find the length of the portion of the "A" road which falls between *X* and *Y* in the scale diagram and find the actual distance from *X* to *Y* along this road in kilometres.

28

Data Representation and Interpretation

By studying this lesson you will be able to;

- construct an ungrouped frequency distribution from given raw data,
- find the mode, median and mean of data in the form of an ungrouped frequency distribution,
- construct a grouped frequency distribution from given raw data,
- identify the modal class and median class from a grouped frequency distribution.

In Grade 8 you learnt how to find the mode, median and mean of given raw data. Do the following review exercise to recall what was learnt.

Review Exercise

1. The ages of the members of a school cricket team (rounded off to the nearest year) are given below.

15, 14, 15, 14, 14, 19, 17, 18, 17, 16, 18

For the above set of data, find the following.

i. The range
ii. The mode
iii. The median
iv. The mean

2. Data collected by a certain weather station on the highest temperature (in degrees Celsius) recorded during each day of the first two weeks of a certain month is given below.

26, 28, 28, 29, 27, 28, 29, 30, 31, 28, 30, 31, 32, 27

For the above set of data, find the following.

i. The range
ii. The mode
iii. The median
iv. The mean

28.1 Ungrouped frequency distribution

To extract the information we require from a given set of raw data, we need to first organize the data in a suitable way. For example, to find a representative value such as the median of a set of raw data, the data needs to be arranged in ascending or descending order.

When there are only a few values, they can easily be arranged in ascending or descending order. However when the number of data is large, arranging them in order and extracting information is not that easy. In such instances it is more appropriate to use tables.

Let us consider such an instance.

The marks obtained in a test by the students of a certain class are given below.

42, 70, 68, 68, 56, 62, 74, 74. 74, 56, 62, 85, 91, 91, 74, 74, 56, 68, 68, 68, 74

This information can be tabulated as follows.

Note: This table can be easily and accurately constructed by using tally marks.

Marks	Tally Marks	Number of Students (Frequency)
42	/	1
56	///	3
62	//	2
68	卌	5
70	/	1
74	卌 /	6
85	/	1
91	//	2

The frequencies are shown in the third column of this table.
Let us first consider what is meant by frequency.

In the above data set, the value 42 occurs once, the value 56 occurs three time, etc. The **frequency** of a value is the number of times that value occurs in the data set.

Accordingly, if we consider the above data set,
the frequency of 42 is 1,
the frequency of 56 is 3,
the frequency of 62 is 2, etc.

An **ungrouped frequency distribution** is a table containing the values of a data set and their respective frequencies.
The following is an ungrouped frequency distribution prepared using the above data set.

Marks	**Number of Students (Frequency)**
42	1
56	3
62	2
68	5
70	1
74	6
85	1
91	2

The mode of the data in an ungrouped frequency distribution

You have learnt in Grade 8 that the **mode** of a data set is the value that is repeated the most in that data set. The largest value in the frequency column of the above table is 6. The value corresponding to the frequency 6 is 74. Therefore the mode of the above data set is 74.

The median of the data in an ungrouped frequency distribution

You have learnt that the **median** of a data set is the value that occurs in the middle when the data is arranged in ascending or descending order.

There are 21 data in the above example. Therefore, when the data is arranged in ascending or descending order, the value that occurs in the middle is the 11th value. Now we need to find out what the 11th value is.

Let us consider how this is done.
Observe from the above table that,

the 1st value is 42,
the 2nd value is 56,
the 3rd value is also 56,
.
.
.
the 6th value is 62.

Accordingly, the 11th value can be found by considering the sums of the values in the frequency column as shown below.

Let us write the sums of the values in the frequency column by the side of the frequency table.

Marks	**Frequency**
42	1
56	3
62	2
68	5
70	1
74	6
85	1
91	2
	21

Sum of the frequencies

1

$3 + 1 = 4$

$2 + 3 + 1 = 6$

$5 + 2 + 3 + 1 = 11$

It can easily be seen by considering the sums of the frequencies in the frequency column that the value in the 11th position is 68.

When there is a large number of data, arranging it in ascending or descending order and identifying the middle value may not be very easy. The following method can be used to identify the middle position (the position of the median).

Note: When the total number of data is an odd number, the middle position is obtained from $\frac{\text{number of data} + 1}{2}$.

The number of data in the above data set $= 21$
When the data is arranged in ascending order,

$$\text{the position where the median is located} = \frac{21+1}{2}$$
$$= 11$$

The value in the 11th position is 68. Therefore, the median of the data set is 68; that is, the median of the marks is 68.

The mean of the data in an ungrouped frequency distribution

You have learnt in Grade 8 that to find the mean of a data set, the sum of all the data values needs to be divided by the number of data values.

Let us see how the mean of the data in an ungrouped frequency distribution is found by considering the above example.

As indicated previously, the value 42 occurs once, the value 56 occurs 3 times, etc. To find the mean, the sum of all the values has to be found.

Let us use a table of the following form to find this sum.

Marks	**Frequency** f	fx
42	1	$42 \times 1 = 42$
56	3	$56\times 3 = 168$
62	2	$62 \times 2 = 124$
68	5	$68 \times 5 = 340$
70	1	$70 \times 1 = 70$
74	6	$74 \times 6 = 444$
85	1	$85 \times 1 = 85$
91	2	$91 \times 2 = 182$
	21	1455

The sum of the data values = 1455

$$\text{The mean of the data set} = \frac{1455}{21}$$
$$= 69.29$$
$$\approx 69 \text{ (rounding off to the nearest whole number)}$$

$\therefore$ the mean of the marks that the students obtained is 69 to the nearest whole number.

Example 1

The masses of 36 grade 3 students of a primary school are given below.
(Mass in kilogrammes)

27	25	20	23	21	26	20	23	21	22	24	25
26	24	23	23	26	24	26	20	24	22	24	25
26	22	23	26	22	24	23	25	24	21	27	27

i. Find the range of the above set of data.
ii. Construct an ungrouped frequency distribution using the above information.
iii. For the above data set, find the following using the frequency distribution.
  (a) Mode
  (b) Median
  (c) Mean

i. The largest value of the data set = 27
The smallest value of the data set = 20
$\therefore$ the range of the data set = 27 − 20
$= \underline{\underline{7}}$

ii.

Mass x (Kg)	**Frequency** f	**Sum of the frequencies**
20	3	3
21	3	6
22	4	10
23	6	16
24	7	23
25	4	27
26	6	33
27	3	36

iii. **a.**The mode of the data set = 24 kg

There are 36 values in this data set. Since 36 is an even number, when this data set is arranged in ascending or descending order, we obtain two middle values. In such a case, the median of the data set is the average of the middle two values.

Let us first find the positions of the middle two values.

Note: When the total number of data is even, the positions of the middle two values are obtained from $\frac{\text{number of data}}{2}$ and $\frac{\text{number of data}}{2} + 1$.

---

b. The positions of the middle two values $= \frac{36}{2}$ and $\frac{36}{2} + 1$

$= 18$ and 19

Therefore the middle two values are in the 18th and 19th positions.

The value in the 18th position = 24
The value in the 19th position = 24

$$\therefore \text{ the median of the data set } = \frac{24 + 24}{2}$$

$$= \frac{48}{2}$$

$$= \underline{\underline{24 \text{ kg}}}$$

c.

Mass x (Kg)	**Frequency** f	$f \times x$
20	3	60
21	3	63
22	4	88
23	6	138
24	7	168
25	4	100
26	6	156
27	3	81
Sum of the data values	36	854

Sum of the data values = 854

Number of data values = 36

$$\therefore \text{mean of the data set} = \frac{854}{36}\text{ kg}$$

$$= \underline{\underline{23.72\text{ kg}}}\text{ (to the nearest second decimal place)}$$

Exercise 28.1

1. The data collected at a certain weather station on the highest temperature (in degrees Celsius) recorded on each day of the month of December in the year 2016 is given below.

28 26 28 28 29 30 28 26 27 27
28 26 25 24 24 25 25 26 27 28
28 27 26 28 27 28 29 30 28 27 27

- **i.** What is the range of this data set?
- **ii.** Construct an ungrouped frequency distribution to find the mode, median and mean of the data set.
- **iii.** Find the mode of the data set using the above constructed frequency distribution.
- **iv.** Find the median of the above set of temperatures.
- **v.** Find the mean of the above set of temperatures.

2. In a certain market, bags containing lime of mass 100 g each are available for sale. The number of limes in each bag is given below.

5 3 4 6 2 3 4 5 3 4 6 5 3 4
4 2 4 3 5 3 3 4 2 5 3 2 4 3

- **i.** What is the range of this data set?
- **ii.** Construct an ungrouped frequency distribution using this data.
- **iii.** Find the mode of the data set.
- **iv.** Find the median of the data set.
- **v.** Find the mean number of limes in a bag (to the nearest whole number).

3. Information on the number of units of electricity consumed daily during a certain period by a certain business establishment is given in the following ungrouped frequency distribution.

Number of units of electricity consumed in a day	8	9	10	11	12	13	14
Number of days	3	5	8	6	4	3	1

i. What is the range of the above data set?
ii. Find the mode of the above data set.
iii. Find the median of the above data set.
iv. Find the mean number of units of electricity consumed per day during the period in which the data was collected.

4. An ungrouped frequency distribution prepared with the information collected on the number of patients who received treatment in the Out Patient Department of a certain hospital each day during a certain period is given below.

Number of patients who received treatment during a day	29	30	31	32	33	34	35
Number of days	2	4	6	8	12	6	2

i. Find the range of this data set.
ii. Find the following for this data set.

  a. Mode
  b. Median
  c. Mean

28.2 Grouped frequency distributions

In this section we will identify what a grouped frequency distribution is, the need for grouped frequency distributions and how they are constructed.

To do this, let us consider the following example.

The marks obtained by a group of students in a certain test is given below.

21	26	28	32	34
36	36	38	39	39
39	40	41	41	41
41	42	45	48	48
52	53	56	66	68
70	75	80	81	83

The highest mark obtained is 83 and the lowest mark obtained is 21.
Therefore the range = 83 – 21 = 62.

Since the range is large and there are many distinct data values, if we try to prepare an ungrouped frequency distribution, we will end up with a fairly long table. In such instances we consider the range of the data set and prepare a table of intervals such that each data value belongs to exactly one of the intervals. These intervals are called **class intervals**. A frequency distribution prepared using class intervals is called a **grouped frequency distribution**.

The following is an example of a grouped frequency distribution.

Class Interval	**Frequency**
10 - 19	3
20 - 29	6
30 - 39	5
40 - 49	2

This distribution has four class intervals.

Any data value which is equal to one of 10, 11, 12, 13, 14, 15, 16, 17, 18 and 19 belongs to the class interval 10 – 19.

Since there are 10 values in the class interval 10 – 19, the **class size (or class width)** is considered to be 10. The class sizes of the other class intervals are defined similarly.

The frequency corresponding to the class interval 10 – 19 is 3. This means that the data set has only 3 values belonging to this class interval.

Now let us consider how a grouped frequency distribution is prepared.

When preparing a grouped frequency distribution, we need to first decide on either the size of the class intervals or the number of class intervals we want to have.

When we have decided on the size of the class intervals, the number of class intervals can be obtained as follows.

- Find the range of the data set.
- Divide the range by the size of a class interval.
- The number of class intervals is the nearest whole number greater or equal to the above obtained value.

Consider the following example which was discussed earlier.

The marks obtained by a group of 30 students in a certain test are given below.

21	26	28	32	34	36	36	38	39	39
39	40	41	41	41	41	42	45	48	48
52	53	56	66	68	70	75	80	81	83

Suppose we want to separate this data set into class intervals of size 10.

Let us first find the number of class intervals.

The largest value of this data set $= 83$
The smallest value of this data set $= 21$
The range $= 83 - 21$
$= 62$

Since we want the size of the class intervals to be 10,

$$\text{the number of class intervals } = \frac{62}{10}$$

$= 6.2$
≈ 7 (when rounded off to the nearest whole number greater than the obtained value)

Accordingly, if we take the class size to be 10, we obtain a grouped frequency distribution with 7 class intervals.

Since the smallest value in the data set is 21, let us prepare the frequency distribution starting with the value 20. The first class interval will then consist of the ten integers 20, 21, 22, 23, 24, 25, 26, 27, 28 and 29. The next class interval will consist of the next 10 integers and so on.

Accordingly, we obtain the following class intervals.

20 - 29

30 - 39

40 - 49

50 - 59

60 - 69

70 - 79

80 - 89

Note: Although we commenced the first class interval from 20, we could have started with the value 21 too (or some other suitable value). If we started with 21, the class intervals would have been 21 – 30, 31 – 40, 41 – 50, etc.

Now, let us find the number of values that fall into each class interval by using tally marks.

Class Interval (Marks)	Tally Marks	Frequency
20 - 29	///	3
30 - 39	~~////~~ ///	8
40 - 49	~~////~~ ////	9
50 - 59	///	3
60 - 69	//	2
70 - 79	//	2
80 - 89	///	3

Note: It is not necessary to include the tally marks column in a frequency distribution.

When we have decided on the number of class intervals, we can find the size of the class intervals (class size) as follows.

- Find the range of the data set by subtracting the smallest value of the data set from the largest value.
- Divide the range by the number of class intervals. (In general, the number of class intervals is taken to be less than 10.)
- Round off the value that is obtained to the nearest whole number greater or equal to it and take this value to be the size of the class intervals.

Let us consider how to construct a grouped frequency distribution with 5 class intervals using the above data set. Let us first find the size of the class intervals.

The largest value of this data set $= 83$

The smallest value of this data set $= 21$

$$\text{The range} = 83 - 21$$
$$= 62$$

Since we require 5 class intervals,

$$\text{the size of each class interval} = \frac{62}{5}$$

$$= 12.4$$

≈ 13 (nearest whole number greater than the obtained value)

Accordingly, we prepare a grouped frequency distribution with 5 class intervals of size 13.

Class Interval	**Frequency**
20 - 32	4
33 - 45	14
46 - 58	5
59 - 71	3
72 - 84	4

As shown above, we can construct grouped frequency distributions according to our requirements, based on the given data set.

Consider the first grouped frequency distribution we constructed. We took 20 - 29 as the first class interval, 30 – 39 as the second class interval, etc. We were able to do this because there were no values between 29 and 30 or between 39 and 40, etc. Observe that this feature is seen in the second grouped frequency distribution we constructed too.

However, if we have a data set consisting of values which are lengths or times or masses, it is necessary to start the second class interval with the value that the first class interval ends, to start the third class interval with the value that the second class interval ends, and so on.

Let us now consider such an example.

The masses of 20 students in a class are given below to the nearest kilogramme.

31	31	31	32	32
32	32	33	33	34
34	34	35	36	36
38	39	39	40	41

Let us construct a grouped frequency distribution with 4 class intervals of size 3 each.

Let us take the first class interval as 30 – 33, the next class interval as 33 – 36, etc.

30 - 33

33 - 36

36 - 39

39 - 42

Here, the second class interval commences with the same value that the first class interval ends. The reason is because the data set consists of masses and masses need not be integral values. For example, we may have students whose masses are 33.2 kg, 33.5 kg, 33.8 kg, etc., which are between 33 kg and 34 kg, or 36.5 kg, 36.9 kg, etc., which are between 36 kg and 37 kg etc. Therefore, in such situations, each class interval needs to commence with the same value that the previous class interval ends (except for the first class interval).

Here, the first class interval ends with 33 and the second class interval commences with the same value 33. A question arises as to which class interval the value 33 belongs. The value 33 can be taken to belong to either one of these two intervals. However, it is important to state the convention that is being used.
In this lesson we will consider the class intervals to be as follows.

Here,

the values greater than 30 but less than or equal to 33 belong to the class interval 30 -33,
the values greater than 33 but less than or equal to 36 belong to the class interval 33 – 36,
the values greater than 36 but less than or equal to 39 belong to the class interval 36 – 39, and
the values greater than 39 but less than or equal to 42 belong to the class interval 39 – 42.

The grouped frequency distribution prepared according to this convention is given below.

Class Interval	Frequency
30 - 33	9
33 - 36	6
36 - 39	3
39 - 42	2

Note: When constructing a grouped frequency distribution, it should be remembered that the class intervals need to be selected by taking the nature of the data into consideration.

Exercise 28.2

1. The data collected by an electricity metre reader on the electricity consumption of each of the households in a certain housing scheme during the month of January 2017 is given below.

63 68 75 54 56 58 85
90 73 63 76 62 69 78
50 74 64 58 88 85 72
71 53 82 68 73 67 75
74 67 69 62 66 74 70
84 72 69 59 67 78 72

Construct a grouped frequency distribution using the above data.

2. The marks obtained in a mathematics test by a group of Grade 9 students of a certain school are given below.

34 27 45 12 63 35 54 29
42 68 73 54 26 11 63 54
33 69 62 38 53 48 63 61
60 44 67 61 79 65 47

i. Find,
(a) the highest mark obtained by a student
(b) the lowest mark obtained by a student.

ii. Find the range of the data set.
iii. For the above data set, construct a grouped frequency distribution with 7 class intervals.

3. The heights (in centimetres) of the Grade 4 students of a certain primary school are given below. Construct a suitable grouped frequency distribution.

124 124 138 125 122 129 122 128 131 127 125 120 125
120 121 125 120 132 127 124 126 130 125 131 122 130
129 128 125 122 133 138 125 123 126 125 135 126 132

28.3 Finding the modal class and median class from a grouped frequency distribution

We learnt how to construct a grouped frequency distribution in the previous section. Now let us consider how the modal class and the median class can be found from a grouped frequency distribution.

When we are given a grouped frequency distribution, we will not be able to identify the mode and the median as the raw data is not available to us. In such situations we consider the modal class and the median class.

The **modal class** is the class interval with the highest frequency. The **median class** is the class interval to which the median belongs.

Example 1

A grouped frequency distribution prepared with the marks obtained by a group of students in a certain test is given below.

From this distribution, find

i. the modal class
ii. the median class.

Marks	Frequency	Sum of the frequencies
10 - 20	3	3
21 - 30	4	7
31 - 40	6	13
$\boxed{41 - 50}$	$\boxed{7}$	⑳
$\boxed{51 - 60}$	$\boxed{11}$	31
61 - 70	4	35

i. Since the highest frequency is 11, the modal class is 51 – 60.

ii. The position of the median of the data set $= \frac{35 + 1}{2}$

$= 18$

The median class is the class interval to which the 18th value belongs. Therefore, the median class is 41 - 50.

Example 2

A grouped frequency distribution prepared using the ages of the employees of a certain establishment is given below.

Find,

i. the modal class

ii. the median class.

Age	Frequency	Sum of the frequencies
20 - 27	3	3
27 - 34	5	8
34 - 41	$\boxed{11}$	19
41 - 58	6	25
48 - 55	3	28

i. The highest frequency $= 11$

$\therefore$ the modal class $= 34 - 41$

ii. The positions of the middle two values $= \frac{28}{2}$ and $\frac{28}{2} + 1$

$= 14$ and 15

The class interval that contains the 14th value $= 34 - 41$
The class interval that contains the 15th value $= 34 - 41$
$\therefore$ the median class 34 - 41.

Exercise 28.3

1. The number of sweep tickets sold each day during the month of March of year 2016 by a certain sweep ticket seller is given below.

380 390 379 402 370 385 397 386 377 405
400 381 390 375 392 384 391 385 387 395
390 393 373 386 378 395 379 396 395 391
373

i. What is the maximum number of sweep tickets that were sold on a day during this period?
ii. What is the minimum number of sweep tickets that were sold on a day during this period?
iii. Find the range of this data set.
iv. Construct a grouped frequency distribution of class size 6.
v. Using the table,

a. find the modal class
b. find the median class.

2. The number of books loaned by a school library during 30 days of the first term of the year 2016 is given below.

27 20 33 37 40 25 15 29 33 32
29 32 25 36 16 35 37 28 34 27
41 36 40 28 27 23 32 33 24 38

i. What is the range of this data set?
ii. Using this data set, construct a grouped frequency distribution consisting of the class intervals 15 – 19, 20 – 24, etc., of class size 5.
iii. Using the table, find the number of days in which 30 or more books have been loaned.
iv. How many days are there in which more than 25 but less than 30 books were loaned?
v. What is the modal class?
vi. To which interval does the median of the number of books loaned each day during this period belong?

Miscellaneous Exercise

1. The ungrouped frequency distribution given below has been prepared with the information collected on the number of coconuts that were plucked from each coconut tree in an estate, during a certain season.

Number of coconuts	Frequency
8	3
10	5
12	8
13	7
14	5
15	2

i. Find the mode of the data set.
ii. Find the median of the data set.
iii. Find the mean number of coconuts plucked from a tree in this estate.

2. The circumferences (in centimetres) of a pile of rubber tree trunks that were purchased to cut planks are given below.

95	112	118	86	103	102	94	98	80	97
87	105	85	103	95	106	98	94	110	102
103	105	90	110	96	100	89	104	98	114
106	98	98	112	86	105	97	107	96	92
115									

i. Prepare a grouped frequency distribution consisting of 8 class intervals.
ii. Find the modal class from this distribution.
iii. Find the median class.